



DAMAGE DETECTION OF STRUCTURES UNDER EARTHQUAKE EXCITATION USING DISCRETE WAVELET ANALYSIS

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ABSTRACT

An effective method for the damage diagnosis of structures under seismic excitation via discrete wavelet transform is proposed in this paper. The proposed method is based on the detection of abrupt changes in seismic vibration responses by the analysis of displacement or velocity responses using wavelet analysis. Also, a wavelet-based method is presented for de-noising of displacement and velocity responses for the damage detection problem. The performance of the proposed method for de-noising and damage detection has been investigated using a benchmark problem provided by the IASC-ASCE Task Group on Structural Health Monitoring and a simulated shear wall model. The obtained results indicate that the proposed method can be provided useful information for the damage occurrence using the seismic response of structures.

Keywords: Damage detection; de-noising; wavelet transform; seismic response; earthquake excitation

1. INTRODUCTION

In the last decade, the researcher's attentions on the detection of structural damage during the service life of engineering structures have been increased. Among numerous methods, approaches that are based on the observation of the dynamic behavior of a structure have been developed [1-4]. Many of these techniques use the identified modal parameters like mode shapes and natural frequencies for structural damage detection and estimation. The identification of alteration in mode shapes and natural frequencies at the damaged system in comparison with the undamaged system is one of the popular methods in the structural damage detection. These changes are often small and measurements are polluted by noise

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making this method an inefficient method.

The wavelet transform is an effective method for precise signal analysis, which overcomes the problems exhibited by other signal processing techniques. Applying wavelet transform for the analysis of damaged structures responses produces satisfying results in the damage identification. Sharp changes in the wavelet coefficients near a damage exhibit the presence of damage. This method is nonparametric and allows the accurate estimation of the time of the sudden change, benefiting from the fine time resolution of the wavelet transform at small scales (i.e. high frequencies). The advantages of damage identification using wavelet analysis are nonparametric and baseline-free data. In addition, it does not depend on the change of the structural frequency, which is sensitive to soil-structure interaction [5].

Wavelet analysis has been applied in the system identification and damage detection. In addition, it has been widely implemented for various purposes, such as the characterization of non-stationary dynamic responses [6, 7]. General overview of damage detection by wavelet analysis may be found in Kim and Melhem [8] and a review on some of the wavelet application such as time-frequency analysis of signals, the fault feature extraction, the denoising and extraction of the weak signals have been done by Peng et al. [9]. In addition, the possibility of applying wavelet transform for the detection of beam cracks has been studied by Sun and Chang [10], Han et al. [11] and Poudel et al. [12]. Damage detection of frame structures via wavelet transform has been analyzed by Ovanesova and Suarez [13] and Hou et al. [14]. Rucka and Wilde [15] proposed a method for estimating damage localization in a beam and a plate by applying continuous wavelet transform. Bayissa and Haritos [16] proposed a new damage identification technique based on the statistical moments of the energy density function of vibration responses in the time-scale domain. Also, Bayissa et al. [17] offered a new damage detection technique using wavelet transform based on the vibration responses of plate. Fan and Qiao [18] developed a two-dimensional continuous wavelet transform-based damage detection algorithm using Dergauss2d wavelet for plate-type structures. Also, a distributed two-dimensional continuous wavelet transform algorithm has been developed by Huang et al. [19]. They used data from discrete sets of nodes and provide spatially continuous variation in the structural response parameters to monitor structural degradation. Recently, a method has been proposed for the detection of crack-like damage in plate structures using discrete wavelet transform by Bagheri et al. [20].

In this study, the event of damages in a structure subjected to an earthquake ground motion is detected, which is related to the numbers of spikes in wavelet results. The proposed methodology is applied to the numerical examples subjected to a real earthquake. The models consist of single and multi degree of freedoms with damping. The efficiency of the presented method is shown based on the obtained results for the damage detection.

2. OVERVIEW ON WAVELET TRANSFORM

Fast Fourier transform is efficient tool for finding the frequency components in a signal. The major disadvantage of fast Fourier transform is that they have only frequency resolution and no time resolution. Therefore, it is not a suitable tool for non-stationary signals. These types of signals can be processed via wavelet transform. It provides a powerful tool to characterize

local features of a signal. The main advantage gained by using the wavelet transform is the ability to perform the local analysis of a signal. Unlike Fourier transform, where the function used as the basis of decomposition is a sinusoidal wave, other basis functions can be selected for wavelet shape according to the features of the signal. The basis function in wavelet analysis is defined by two parameters named scale and translation. This property leads to a multi-resolution representation for non-stationary signals. As mentioned before, a basis function or mother wavelet is used in wavelet transform.

The wavelet transform is a transformation that decomposes a function $S(t)$ into a superposition of the elementary function $\psi_{a,b}(t)$ derived from an mother wavelet $\psi(t)$ by scaling and translating, as defined below:

$$\psi_{a,b}(t) = |a|^{-\frac{1}{2}} \psi\left(\frac{t-b}{a}\right) \quad (1)$$

where, a and b are scale and translation parameters, respectively.

The continuous wavelet transform of the signal $S(t)$ is defined as [21]:

$$W_{\psi}(a,b) = \int_{-\infty}^{+\infty} S(t) \psi^*\left(\frac{t-b}{a}\right) dt \quad (2)$$

where, ψ^* is the complex conjugate of ψ and $W_{\psi}(a,b)$ is called the wavelet coefficient for the wavelet $\psi_{a,b}(t)$.

In signal processing, a discrete version of wavelet transform is often used by discretizing the scale parameter a , and the translation parameter b . In general, the procedure becomes much more efficient if dyadic values of a and b are used, i.e.:

$$a = 2^{-j}, \quad b = 2^{-j} k \quad (3)$$

where j and k are integers.

The corresponding discretized wavelet $\psi_{j,k}(t)$ is as follows:

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad (4)$$

Equation (2) can be expressed as:

$$W_{\psi}(a,b) = \langle S(t), \psi_{a,b}^*(t) \rangle \quad (5)$$

Thus, continuous wavelet transform is a collection of inner products of a signal and the translated and dilated wavelets. In this paper, the mother wavelets $\psi(t)$ are the Daubechies wavelet, Biorthogonal Spline and Coiflet wavelet.

3. DAMAGE IDENTIFICATION METHOD

In the linear range, the equation of motion of a excited structure by a seismic input can be

express as:

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = -\mathbf{M}\mathbf{\Gamma}\ddot{x}_g(t) \quad (6)$$

where, \mathbf{M} , \mathbf{C} , and \mathbf{K} are mass, damping, and stiffness matrices of the structure, respectively; \mathbf{X} is the vector of displacements; $\mathbf{\Gamma}$ describes the influence of the earthquake excitation; \ddot{x}_g is earthquake acceleration vector.

Defining the state vector $\mathbf{Z}(t) = [\mathbf{X}(t) \ \dot{\mathbf{X}}(t)]^T$, the Eq. (6) can be written in state space form as:

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}\ddot{x}_g(t) \quad (7)$$

where \mathbf{A} is system matrix; and \mathbf{B} is a influence vector for the earthquake excitation. The system matrix and excitation influence vector are given by:

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}; \quad (8)$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{\Gamma} \end{bmatrix}$$

In the proposed method, the aim is to obtain the time of damage occurrence in the structure under seismic excitation. For this reason, the proposed approach employs the decomposing capabilities of wavelet analysis on structural responses. The displacement or velocity responses $r(t)$ can be represented by approximations and details of discrete wavelet transform. Therefore, the objective is to decompose the response in this step. The detail of the response at level j is defined as:

$$D_j(t) = \sum_{k \in Z} cD_{j,k}(k) \psi_{j,k}(t) \quad (9)$$

where Z is the set of positive integers, and $cD_{j,k}$ is wavelet coefficients at level j which is defined as:

$$cD_j(k) = \int_{-\infty}^{\infty} r(t) \psi_{j,k}(t) dt \quad (10)$$

The approximation of the response at level j is defined as:

$$A_j(t) = \sum_{k=-\infty}^{\infty} cA_j(k) \phi_{j,k}(t) \quad (11)$$

where ϕ is the scaling function and $cA_{j,k}$ is scaling coefficients at level j which is defined as:

$$cA_j(k) = \int_{-\infty}^{\infty} r(t) \phi_{j,k}(t) dt \quad (12)$$

In addition, the response can be expressed by:

$$r(t) = A_j(t) + \sum_{j \leq J} D_j(t) \quad (13)$$

Finally, the peak values of the details of the response can be expressed the time of damage occurrence in the structure.

4. DE-NOISING METHOD OF RESPONSES

The measured responses of structures always contain noise. The existence of noise may have an appreciable influence on the accuracy of damage detection. A systematic investigation of the effects of noise on the performance of damage identification method is yet to be done.

In this study, the de-noising procedure using the wavelet transform involves three steps. The basic version of the procedure follows the steps described below:

1. Decompose: Compute the wavelet decomposition of the measured response at level J .
2. Threshold detail coefficients: For each level from l to J , select a threshold and apply soft thresholding to the detail coefficients.
3. Reconstruct: Compute wavelet reconstruction using the original approximation coefficients of level J and the modified detail coefficients of levels from l to J .

5. NUMERICAL STUDIES

In the following section, a numerical example namely a concrete shear wall and the first phase of IASC-ASCE benchmark structure are show for the feasibility of proposed method.

5.1. Concrete shear wall

This numerical example is a concrete shear wall with the height $H=4$ m, length $L= 3$ m and thickness $t=0.15$ m. For the considered shear wall, the material properties include Young's modulus of $E=20$ GPa, poisson's ratio $\nu=0.2$, damping ratio $\zeta=0.05$ and lump mass in the top of the wall $M=1 \times 10^6$ kg. The lateral stiffness K of the shear wall is calculated as follows:

$$I_x = \frac{t \times L^3}{12}, \quad A_v = \frac{5}{6} t \times L, \quad G = \frac{E}{2(1+\nu)}$$

$$K = \frac{1}{\frac{H^3}{12EI_x} + \frac{H}{GA_v}} \quad (14)$$

where I_x , A_v and G are the second moment of inertia and shear area and shear modulus, respectively.

Damage is simulated by reducing the stiffness of shera wall. There are two damage cases: the damage for the first case is a 30% reduction of stiffness in 2.5th second of earthquake accelerogram and 20% and 40% reduction of stiffness in 2.5th and 10th second of earthquake accelerogram for the second case, respectively.

To illustrate the potential application of the proposed method, El-Centro (1940) earthquake accelerogram has been considered for the numerical simulation as shown in Figure 1. Figures 2 and 3 show the simulated response of the shear wall under El-Centro (1940) earthquake accelerogram for the first and second damage cases, respectively.

In this paper, the displacement and velocity responses have been used as a signal for damage identification that has shown the properties of structures better. Because, it is directly relate to the properties of material that are used in structures such as stiffness and damping. In this way, using wavelet analysis can provide a good procedure to detect abrupt changes in the response by decomposing, windowing and zooming. In this paper, all of the responses have been decomposed in 1 level with different mother wavelets such as *db20*, *bior5.5* and *coif4*.

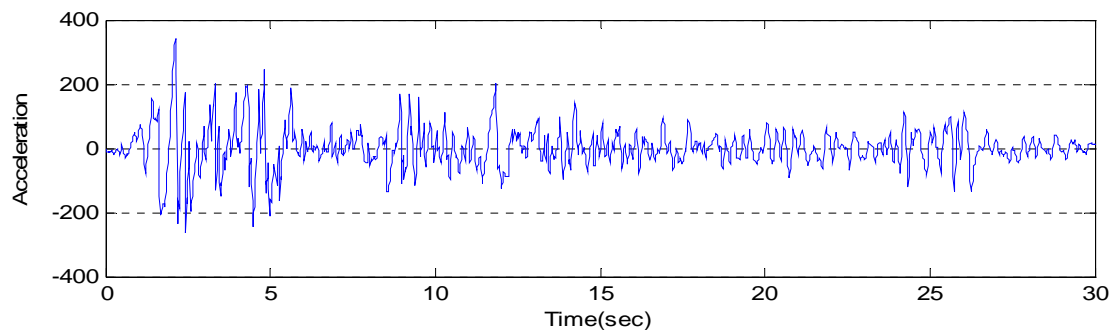


Figure 1. El-Centro (1940) earthquake accelerogram

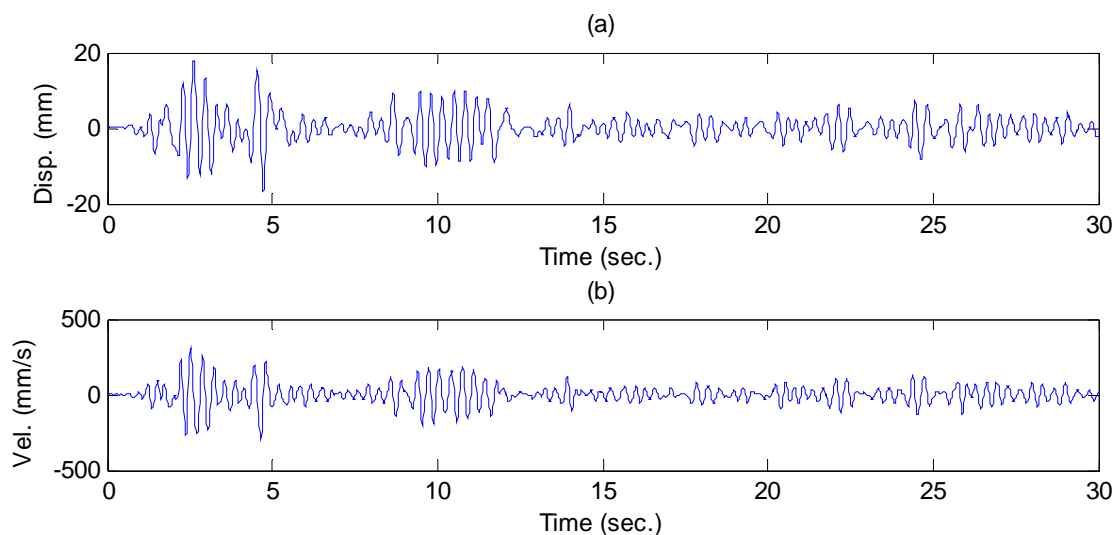


Figure 2. Displacement and velocity responses for the first damage case under El-Centro (1940) earthquake

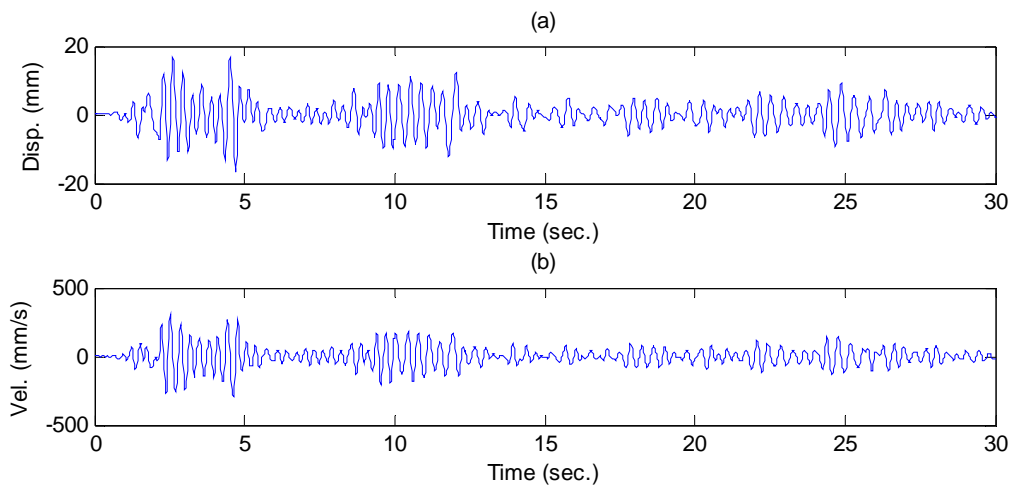


Figure 3. Displacement and velocity responses for the second damage case under El-Centro (1940) earthquake

Figures 4 to 6 show the obtained results for the first damage case under the El-Centro (1940) earthquake using different mother wavelets. For the second damage case, the obtained results are shown in Figures 7 to 9. Spikes in details are corresponding to abrupt changes in the response that might be associated with structural damage. In the other word, damages of the structure subjected to the earthquake ground motion are related to the number of spikes in the wavelet results.

To be more suited with the real cases, another examination has been performed in which the structural responses with 5% noise are utilized to damage identification considering the same damage cases mentioned before. Figures 10-12 and 13-15 illustrate that the proposed method is robust and promising in detecting various damage cases with noise for the first and second damage cases; respectively.

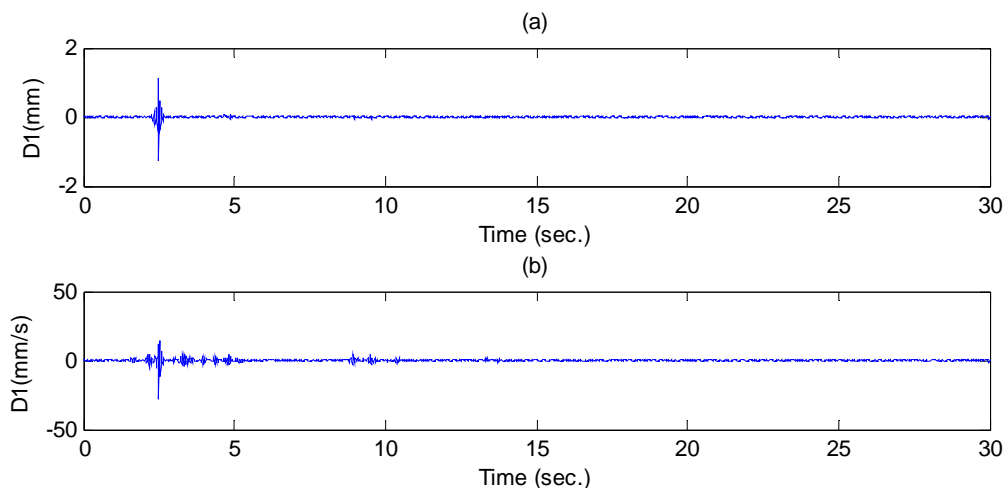


Figure 4. The obtained results of damage detection for the first damage case under El-Centro (1940) earthquake using Daubechies wavelet

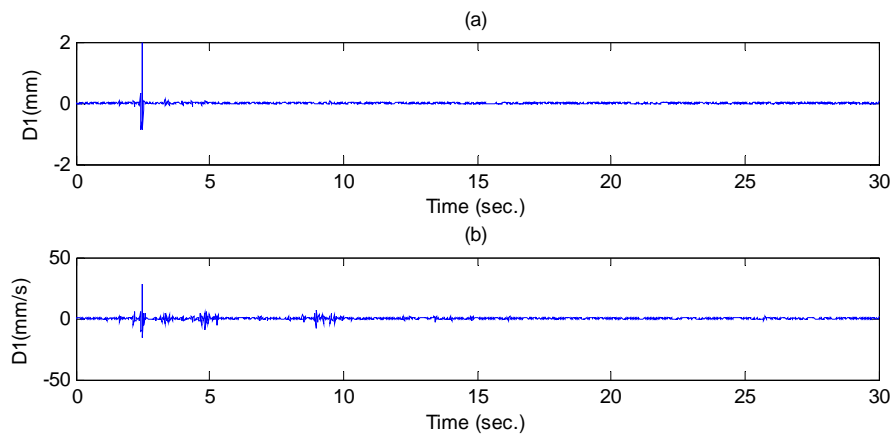


Figure 5. The obtained results of damage detection for the first damage case under El-Centro (1940) earthquake using Biorthogonal spline wavelet

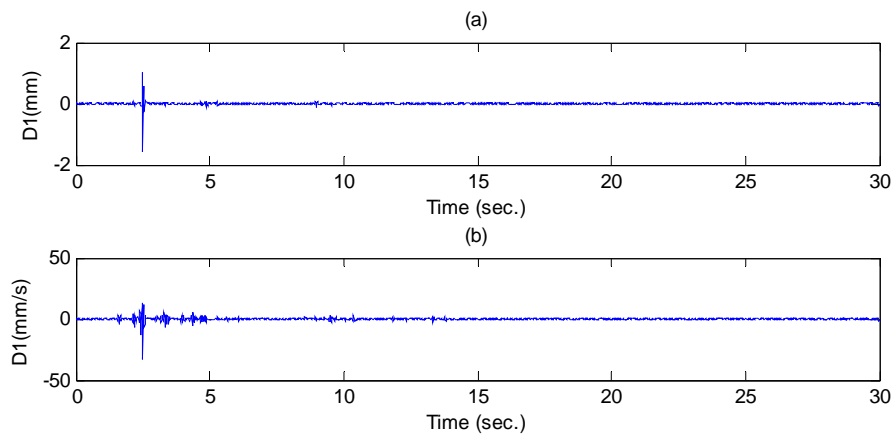


Figure 6. The obtained results of damage detection for the first damage case under El-Centro (1940) earthquake using Coiflet wavelet

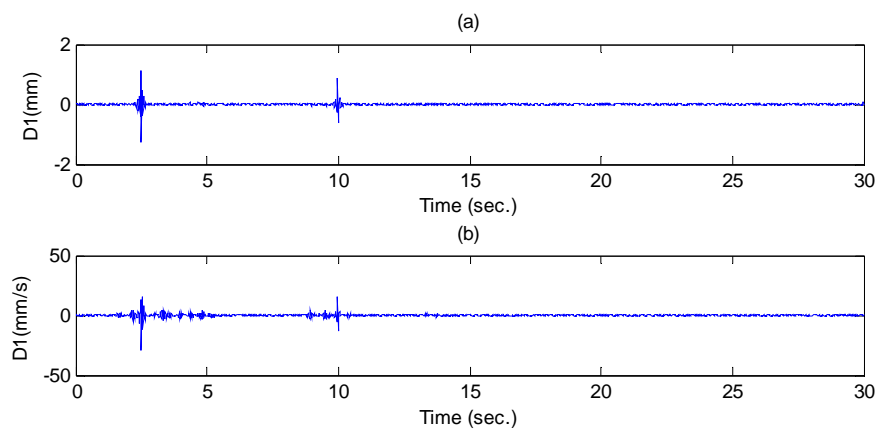


Figure 7. The obtained results of damage detection for the second damage case under El-Centro (1940) earthquake using Daubechies wavelet

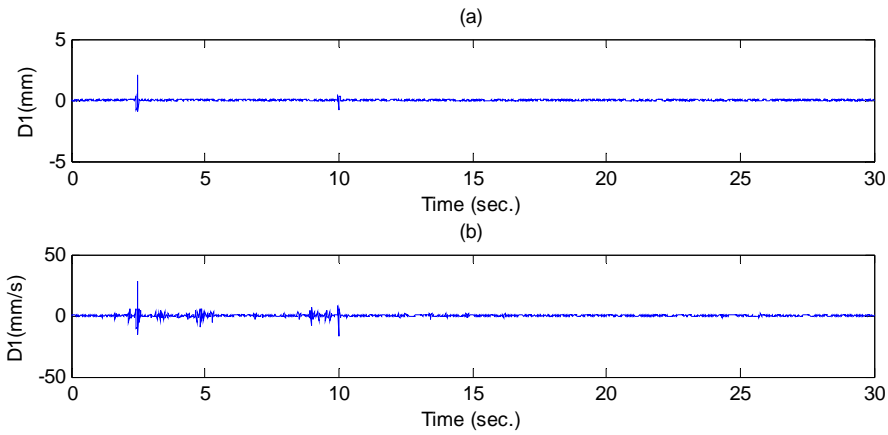


Figure 8. The obtained results of damage detection for the second damage case under El-Centro (1940) earthquake using Biorthogonal spline wavelet

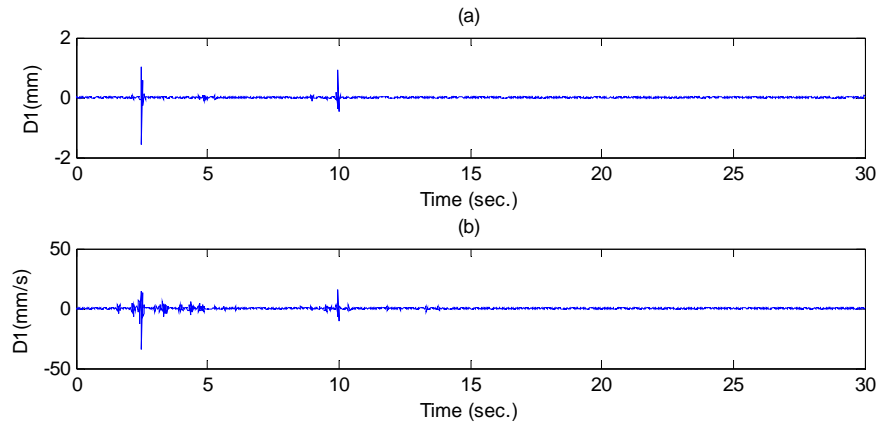


Figure 9. The obtained results of damage detection for the second damage case under El-Centro (1940) earthquake using Coiflet wavelet

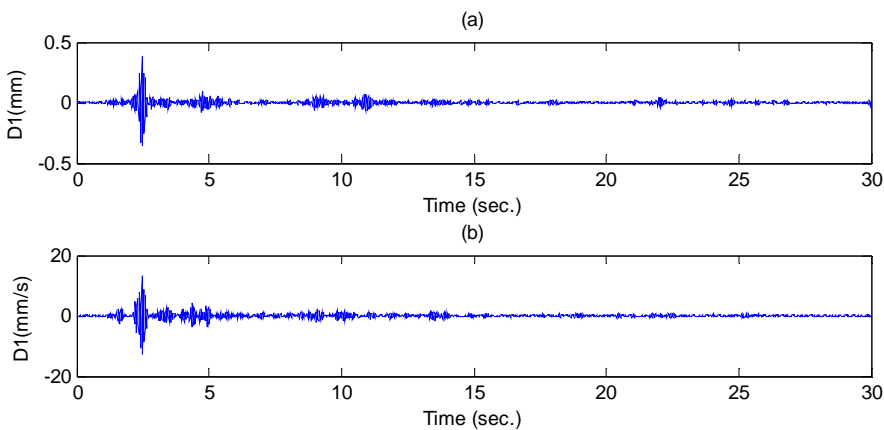


Figure 10. The obtained results of damage detection for the first damage case based on 5% noise-corrupted responses signals using Daubechies wavelet

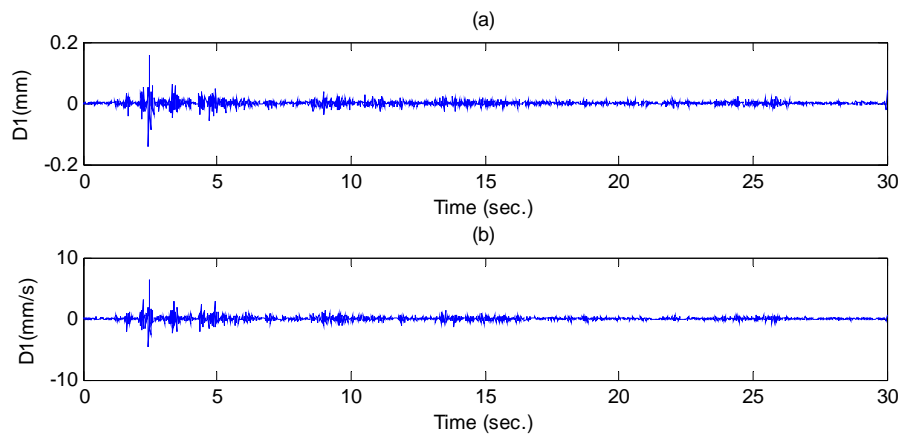


Figure 11. The obtained results of damage detection for the first damage case based on 5% noise-corrupted responses signals using Biorthogonal spline wavelet

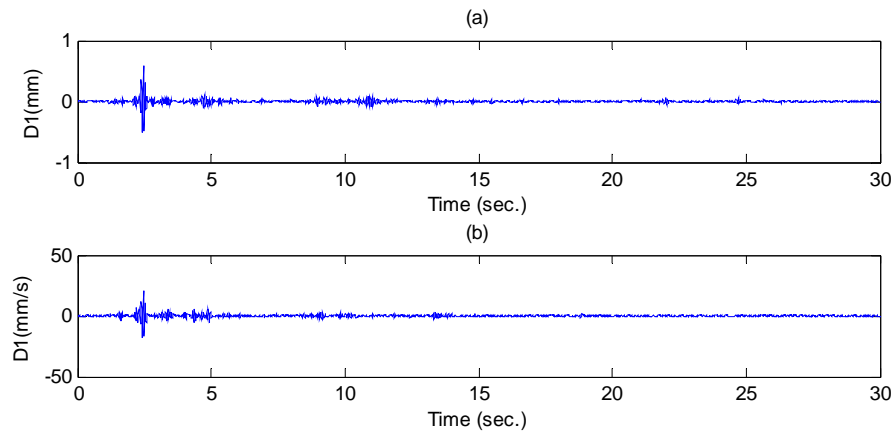


Figure 12. The obtained results of damage detection for the first damage case based on 5% noise-corrupted responses signals using Coiflet wavelet

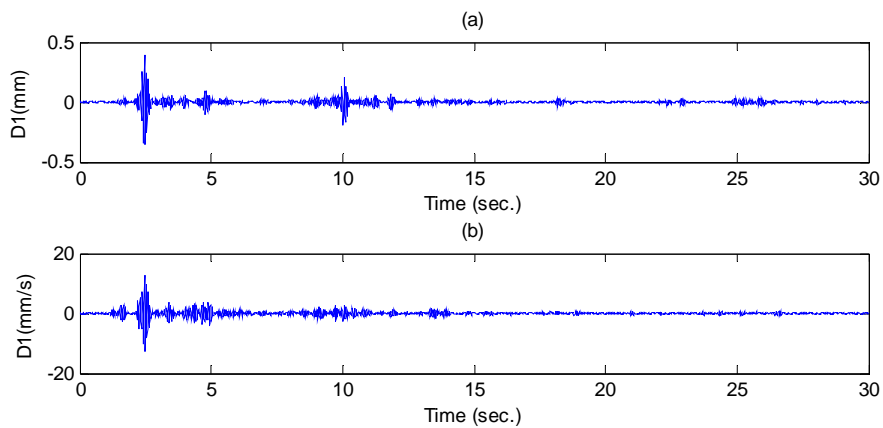


Figure 13. The obtained results of damage detection for the second damage case based on 5% noise-corrupted responses signals using Daubechies wavelet

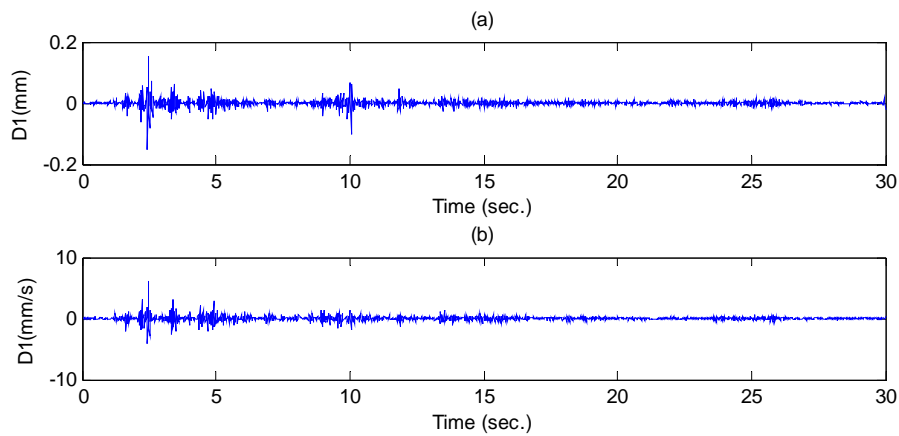


Figure 14. The obtained results of damage detection for the second damage case based on 5% noise-corrupted responses signals using Biorthogonal spline wavelet

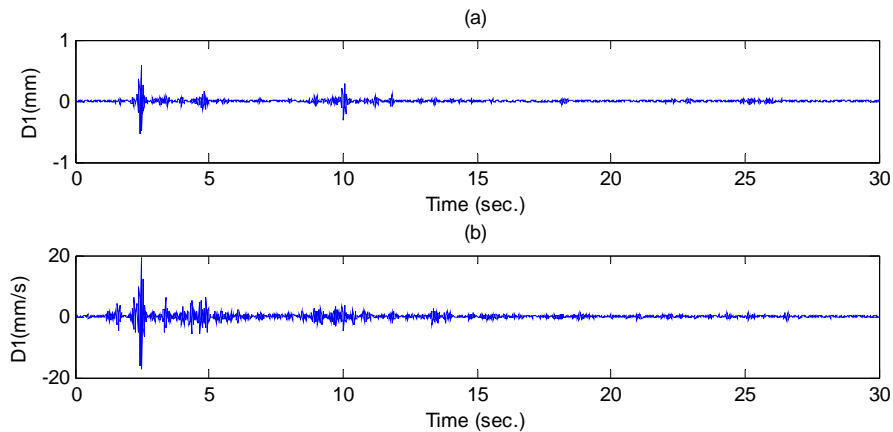


Figure 15. The obtained results of damage detection for the second damage case based on 5% noise-corrupted responses signals using Coiflet wavelet

5.2. IASC-ASCE benchmark structure

The benchmark structure is a four-story steel frame, two-bay by two-bay and quarter-scale model structure constructed in the Earthquake Engineering Research Laboratory at the University of British Columbia. Figure 16 shows the geometry of the benchmark structure. Details of the first phase of IASC-ASCE benchmark problem has been presented by Johnson et al. [22] also are available on IASC-ASCE Structural Health Monitoring Task Group web site [23]. The proposed method for the damage identification has been applied to case 1 of this phase benchmark problem, and the finite element model of the 12 DOF shear building model has been used. In this case, the three following damage patterns have been considered:

1. All of the braces of the first story are broken
2. One brace of the first story is broken
3. 1/3 of area of one brace at the first story is cut

For each damage case, the mass and horizontal story stiffnesses are illustrated in Table 1. In all of damage cases, the reduction of stiffness is at 10th second of earthquake accelerogram. In this example, El-Centro (1940) earthquake accelerogram has been applied for numerical simulation.

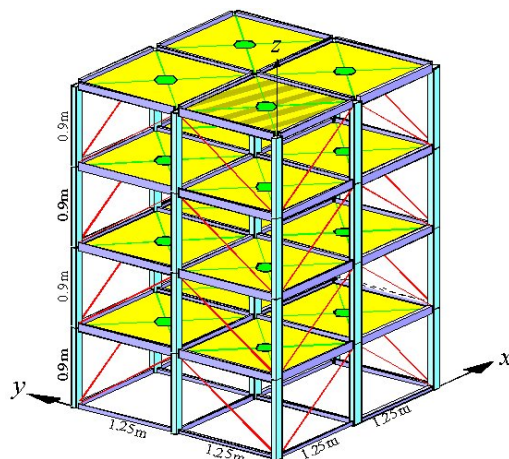


Figure 16. Geometry of the IASC-ASCE benchmark structure [23]

Table 1: Mass and horizontal story stiffness (MN/m) of undamaged and damaged model

Story	DOF	Mass (kg)	Undamaged Stiffness	Pattern (1) stiffness	Pattern (2) stiffness	Pattern (3) stiffness
1	x	3452.4	106.60	58.37	106.6	106.6
1	y	3452.4	67.90	19.67	55.84	63.88
1	θ_z	3819.4	232.00	81.32	213.12	225.71
2	x	2652.4	106.60	106.60	106.60	106.60
2	y	2652.4	67.90	67.90	67.90	67.90
2	θ_z	2986.1	232.00	232.00	232.00	232.00
3	x	2652.4	106.60	106.60	106.60	106.60
3	y	2652.4	67.90	67.90	67.90	67.90
3	θ_z	2986.1	232.00	232.00	232.00	232.00
4	x	1809.9	106.60	106.60	106.60	106.60
4	y	1809.9	67.90	67.90	67.90	67.90
4	θ_z	2056.9	232.00	232.00	232.00	232.00

Damage in the structures can be determined using the proposed method which employs the wavelet analysis. The obtained results of the application of the proposed method to the benchmark structure for the three damage patterns are shown in Figures 17 to 19. The structural damage has been detected within different wavelet mothers. The results reveal that there is a good agreement between the actual and estimated damage of the benchmark structure. Lastly, it can be concluded that this damage detection method is much more sensitive to the time of damage occurrence in the structure. This is due to the use of wavelet analysis in the presented method for signal processing.

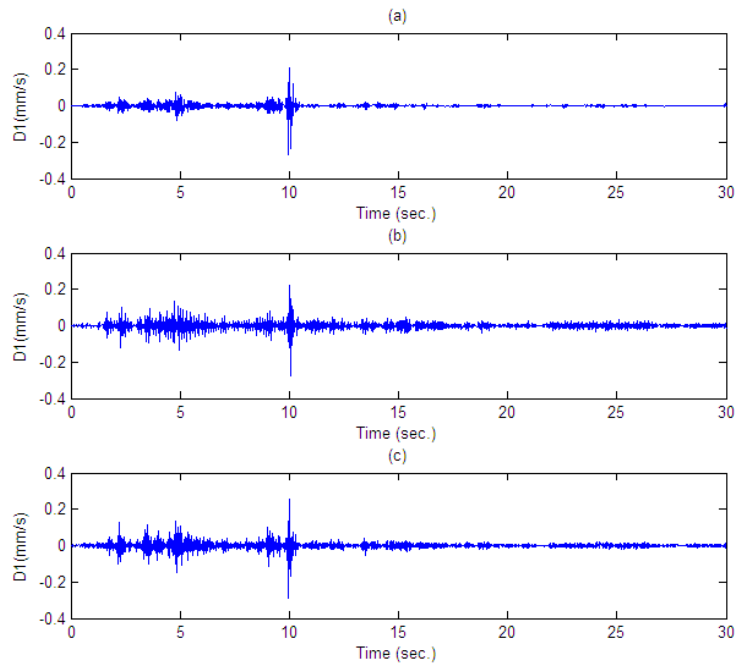


Figure 17. The obtained results of damage detection for the first damage case of the IASC-ASCE benchmark structure using: (a) Daubechies wavelet, (b) Biorthogonal spline wavelet and (c) Coiflet wavelet

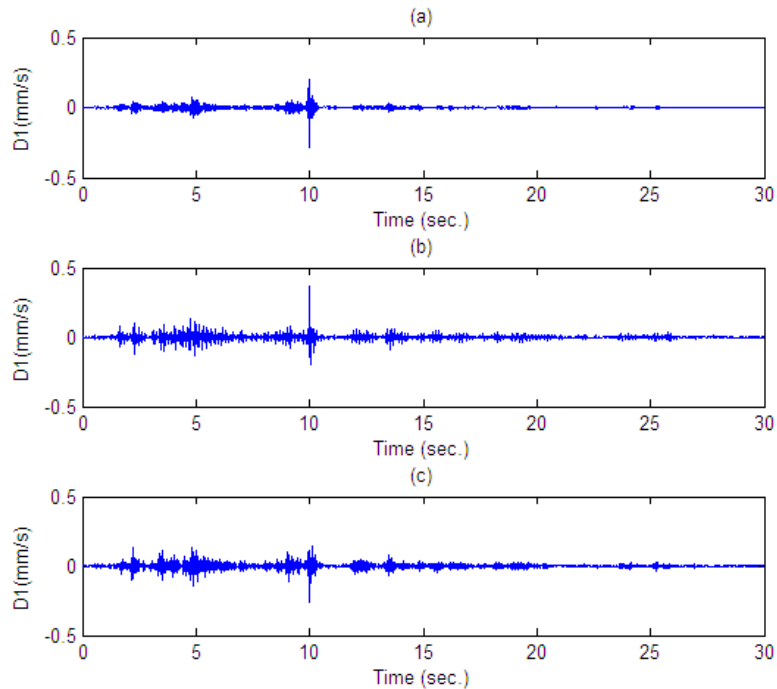


Figure 18. The obtained results of damage detection for the second damage case of the IASC-ASCE benchmark structure using: (a) Daubechies wavelet, (b) Biorthogonal spline wavelet and (c) Coiflet wavelet

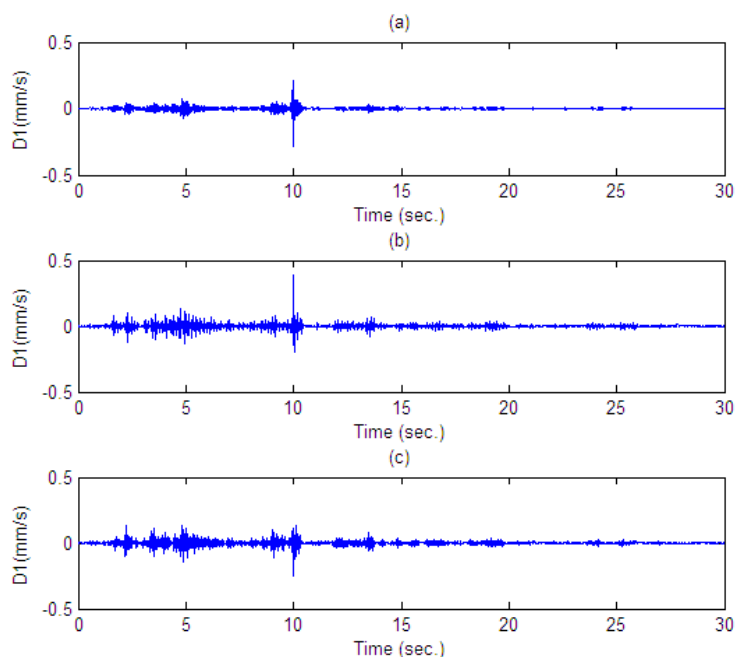


Figure 19. The obtained results of damage detection for the third damage case of the IASC-ASCE benchmark structure using: (a) Daubechies wavelet, (b) Biorthogonal spline wavelet and (c) Coiflet wavelet

6. CONCLUSIONS

In this study a new method presented for damage detection of structures under seismic excitation using the wavelet analysis. The present method is aimed at damage diagnosis of the structures based on decomposition problem. In addition, a wavelet-based approach is proposed for de-noising of displacement and velocity responses for damage detection problem.

In the illustrative examples, the proposed method is applied to a numerical example namely a concrete shear wall and the first phase of IASC-ASCE benchmark structure. The results obtained indicate that the damage detection of the structures based on the displacement and velocity responses has shown a good clarity and precision. In addition, changes in the seismic vibration response signal are directly related to the changes in the properties of structures such as stiffness, damping and mass. Abrupt changes of these signals in the wavelet analysis show the damage of structure. By signal processing, we can detect the damage under seismic excitation but it is related to the nature of earthquake waves and the magnitude of excitation. This method helps for online monitoring of infrastructures to detect the damages that occur in structures.

REFERENCES

1. Doebling SW, Farrar CR, Prime MB, Schevitz DW. Damage identification and health

monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review, Technical Report LA-13070-MS, Los Alamos National Laboratory, Los Alamos, 1996.

2. Bagheri A, Ghodrati Amiri G, Seyed Razaghi SA. Vibration-based damage identification of plate structures via curvelet transform, *Journal of Sound and Vibration*, **327**(2009) 593–603.
3. Ghodrati Amiri G, Seyed Razaghi SA, Bagheri A. Damage detection in plates based on pattern search and genetic algorithms, *Smart Structures and Systems*, No. 2, **7**(2011) 117–32.
4. Brincker R, Andersen P, Cantieni R. Identification and level I damage detection of the Z24 highway bridge, *Experimental Techniques*, No. 6, **25**(2001) 51–57.
5. Todorovska MI, Trifunac MD. Structural health monitoring by detection of abrupt changes in response using wavelets: application to a 6-story RC building damaged by an earthquake, Technical Memorandum of Public Works Research Institute, No. 3983, 2005, 245–65.
6. Brenner MJ. Non-stationary dynamics data analysis with wavelet-SVD filtering, *Mechanical Systems and Signal Processing*, **17**(2003) 765–86.
7. Kitada Y. Identification of non-linear dynamic systems using wavelets, *Journal of Engineering Mechanics, ASCE*, **124**(1998) 152–57.
8. Kim H, Melhem H. damage detection of structures by wavelet analysis, *Engineering Structures*, **26**(2004) 347–62.
9. Peng ZK, Chu FL. Application of the wavelet transform in machine condition monitoring and fault diagnostics: a review with bibliography, *Mechanical Systems and Signal Processing*, **18**(2004) 199–221.
10. Sun Z, Chang CC. Statistical wavelet-based method for structural health monitoring, *Journal of Structural Engineering, ASCE*, No. 7, **130**(2004) 1055–62.
11. Han J, Ren W, Sun Z. Wavelet packet based damage identification of beam structures, *International Journal of Solids and Structures*, **42**(2005) 6610–27.
12. Poudel UP, Fu G, Ye J. Wavelet transformation of mode shape difference function for structural damage location identification, *Earthquake Engineering and Structural Dynamics*, **36**(2007) 1089–107.
13. Ovanesova AV, Suarez LE. Application of wavelet transform to damage detection in frame structures, *Engineering Structures*, **26**(2004) 39–49.
14. Hou Z, Noori M, Amand RS. Wavelet-based approach for structural damage detection, *Journal of Engineering Mechanics, ASCE*, No. 7, **126**(2000) 677–83.
15. Rucka M, Wilde K. Application of continuous wavelet transform in vibration based damage detection method for beams and plates, *Journal of Sound and Vibration*, **297**(2006) 536–50.
16. Bayissa, W.L. and Haritos, N.: Structural damage identification in plates using spectral strain energy analysis, *Journal of Sound and Vibration*, **307**(2007) 226–49.
17. Bayissa WL, Haritos N, Thelandersson S. Vibration-based structural damage identification using wavelet transform, *Mechanical Systems and Signal Processing*, **22**(2008) 1194–215.
18. Fan W, Qiao P. A 2-D continuous wavelet transform of mode shape data for damage

- detection of plate structures, *International Journal of Solids and Structures*, **46**(2009) 4379–95.
19. Huang Y, Meyer D, Nemat-Nasser S. Damage detection with spatially distributed 2D Continuous Wavelet Transform, *Mechanics of Materials*, **41**(2009) 1096–107.
 20. Bagheri A, Ghodrati Amiri G, Khorasani M, Bakhshi H. Structural damage identification of plates based on modal data using 2D discrete wavelet transform, *Structural Engineering and Mechanics*, No. 1, **40**(2001) 341–56.
 21. Daubechies I. Ten lectures on wavelets, CBMS-NSF Conference Series in applied Mathematics, Montpelier, Vermont, 1992.
 22. Johnson EA, Lam HF, Katafygiotis LS, Beck JL. Phase I IASC-ASCE structural health monitoring benchmark problem using simulated data, *Journal of Engineering Mechanics, ASCE*, **130**(2004) 3-15.
 23. <http://wusceel.cive.wustl.edu/asce.shm>