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Experimental validation of a theoretical model for flexural modulus of elasticity of thin cement composite

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ABSTRACT

Experimental and analytical investigations for the modulus of elasticity of thin cement composite composed of mesh and mortar are demonstrated. Based on the analyses and experimental data, new equations for the modulus of elasticity of thin cement composite are proposed. It is observed that the flexural modulus of elasticity of thin cement composite depends on the elastic modulus of mortar and some factor of the difference of elastic modulus of mesh and mortar. Results obtained by using the proposed equations are compared to those of the available equations. It has been found that the newly developed equations give relatively conservative results as compared to the typically used ones. A comparison between the analytical and experimental findings further indicates that there is a good agreement between the analytical and experimental results.

1. Introduction

Extensive experimental and analytical studies have been undertaken during the last couple of decades to establish the fundamental mechanical properties of thin cement composite [1,2]. The application of finite element method for analyzing thin cement composite has been investigated by Prakhya and Adidam [3] and Hossain and Hasegawa [4]. They have reported the modeling technique and load–deflection behavior of thin cement composites containing square and chicken meshes. Rao investigated the load deformation data in the form of stress–strain relationships of thin cement composites reinforced with chicken meshes under uniaxial compression [5]. He concluded that the stress–strain relationships under compression possess non-linearity at the initial and final loadings with the linearity at mid-section. The properties of impact damage of thin cement composites were obtained by the lateral single impact tests undertaken by Kobayashi et al. [6]. Later, bending behavior of thin cement composites has been studied by Ghavami et al. [7] and Naaman [8].

In spite of the available technical information, very little is known on the mechanical properties such as modulus of elasticity in flexure for various types of reinforcements in cement composite except for the incomplete research works that can be found by Al-Rifaie and Aziz on the equation of Young's modulus in the secondary direction of hexagonal mesh reinforced cement composites [9]. They have demonstrated that the composite Young's modulus in the secondary direction equals to the some fraction of Young's modulus of mortar minus ratio of mesh wires along a potential crack to the total area in percent plus some numerical values. The demerit of this equation is that it has three components of different units. Moreover, it can be used to calculate the Young's modulus in the secondary direction only. The modulus of elasticity in the longitudinal direction that represents a significant property of cement composite in flexural design of structures has not been fairly checked.

A thorough investigation on thin cement composite is, therefore, necessary to understand the flexural behavior of cement composites containing various kinds of reinforcements, and to develop a design equation for the modulus of elasticity in flexure. The purpose of this research is to investigate the modulus of elasticity of cement composites with various types of reinforcements commonly used in practice and readily available in local markets. Along with the theoretical study, an experimental investigation was also carried out to examine the validity of the results presented in this paper.

2. Research significance

A new design equation for flexural modulus of elasticity of thin cement composites reinforced with different types of meshes is proposed in this paper. A flexural section of thin cement composite is theoretically analyzed, and an experimental investigation is carried out for validation of the proposed equation. The paper also depicts the required mesh factor (MF) for the sake of ease in the design process when a flexural composite is reinforced with

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Nomenclature

ASTM	American Society for Testing and Materials	dy	infinitesimal layer in mortar and mesh portions
D	center-to-center distance of mesh wires	t	thickness of cement composite
E_{com}	modulus of elasticity of composite	y	distance from the neutral axis to dA
E_m	modulus of elasticity of mortar	y_0	represents the mortar portion
E_r	modulus of elasticity of reinforcement	φ	denotes type of mesh
I_c	moment of inertia of the composite flexural section	ε_r	indicates the part of effective reinforcement
JIS	Japan Industrial Standards	ε_r	strain in the mesh layer
JSCE	Japan Society of Civil Engineers	ε_m	strain in the mortar layer
L	span length	Δdx	deformation a layer of length dx
MF	mesh factor	ε	strain in any layer of length dx
N_L	number of layers	σ_r	stresses developed in mesh layers
P	applied third point load	σ_m	stresses developed in mortar layers
R_r	effective reinforcement	θ	angle of the mesh wires to the panel axis
a	proportional constant	Δy	represents the mesh portion
b	width of the cement composite	β	indicates the part of mortar
d	diameter of wire	δ	deflection at the center
dA	infinitesimal area expressed as bdy	ζ	indicates uncertainty

different meshes. It is anticipated that this study dealing with the development of flexural modulus of elasticity of thin cement composites will be useful for the construction of composite structures.

3. Typical equations for composite modulus of elasticity

The modulus of elasticity ($E_{com.}$) of a composite member consisting of different materials under uniaxial loading is usually expressed as follows

$$E_{com.} = E_m + R_r(E_r - E_m) \tag{1}$$

where R_r is the effective reinforcement, E_m is the modulus of elasticity of mortar and E_r is the modulus of elasticity of reinforcement.

The effective reinforcement (R_r) given in Eq. (1) is defined as the ratio of the area of mesh wires in the longitudinal direction to the total area of specimen in the same direction. Here, the effective reinforcement (R_r) for cement composite element containing square mesh (Fig. 1) in the longitudinal direction is derived as:

$$R_r = \frac{25\pi\, d^2 N_L}{Dt} \tag{2}$$

where d is the diameter of wire, N_L is the number of layers, t is the thickness of cement composite and D is the center-to-center distance of mesh wires. The numerical value 25 is a conversion factor for expressing the R_r in percentage.

For cement composite reinforced with chicken mesh, the effective reinforcement (R_r) can be expressed as:

$$R_r = \frac{25\pi\, d^2 N_L \cos\theta}{Dt} \tag{3}$$

where θ is the angle of the mesh wires to the panel axis and has a value of 59.53° for the mesh used in the present research work.

4. Derivation of composite modulus of elasticity in flexure

For a flexural section of cement composite containing four mortar layers and two mesh layers as shown in Fig. 3, the equation of equilibrium in the elastic range can be written as follows:

$$M = 2\int_0^{y_0} y\sigma_m dA + \int_{y_0}^{y_0+\Delta y} y\sigma_r dA + \int_{y_0+\Delta y}^{t/2} y\sigma_m dA \tag{4}$$

where t is the thickness of the flexural section, y_0 and Δy represent the mortar and mesh portions, respectively; σ_m and σ_r are stresses developed in mortar and mesh layers, respectively, and y is the distance from the neutral axis to the area dA. The dA can be expressed as bdy where b is the width of the flexural section and dy is the height of strip taken in the mortar portion and mesh portion. The strain (ε) in any layer of length dx at distance y from the neutral axis can be written as follows

$$\varepsilon = \frac{\Delta dx}{dx} = a \cdot y \tag{5}$$

where Δdx is the deformation of the layer and a is the proportional constant. By using the Hook's law, the strain (ε) in any layer given in Eq. (5) can be written as follows

$$\varepsilon = \frac{\sigma_m}{E_m} = \frac{\sigma_r}{E_r} \tag{6}$$

The strains in the mortar (ε_m) and mesh (ε_r) layers are same, i.e. $\varepsilon_r = \varepsilon_m$ but the stresses in the mortar (σ_m) and mesh (σ_r) layers are not same, i.e. $\sigma_m \neq \sigma_r$ (Fig. 3).

Substituting Eqs. (5) and (6) into Eq. (4), the following equation is obtained.

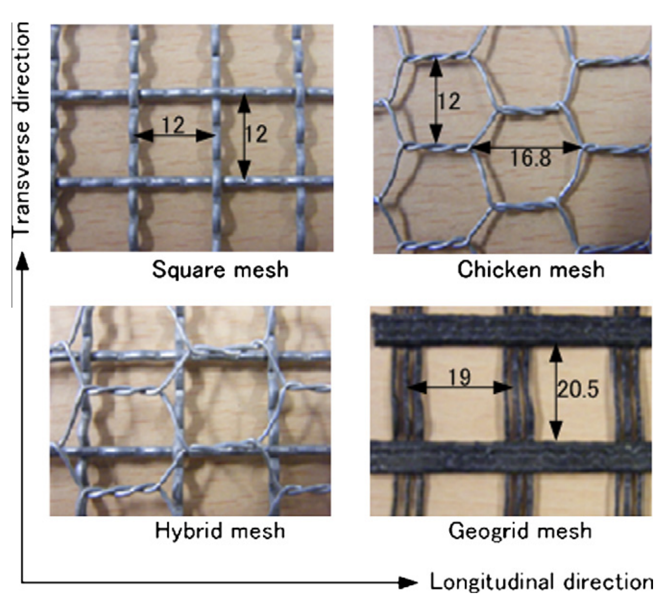

Fig. 1. Various types of mesh (all dimensions in mm).

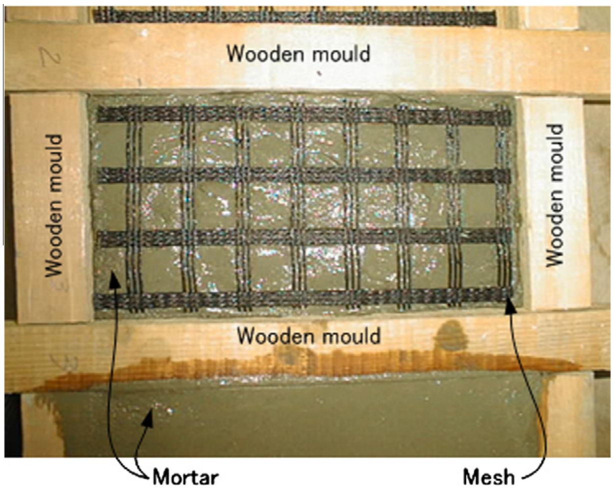

Fig. 2. Photograph showing placement of mortar and mesh layers.

$$M = 2b\left[\int_0^{y_0} E_m a y^2 dy + \int_{y_0}^{y_0+\Delta y} E_r a y^2 dy + \int_{y_0+\Delta y}^{t/2} E_m a y^2 dy\right] \quad (7)$$

where E_m and E_r are the moduli of elasticity of mortar and mesh layers, respectively.

After integration, the Eq. (7) with some calculations takes the following form:

$$M = a\left[E_m\frac{bt^3}{12} + 3(E_r - E_m)\alpha\beta^2\frac{bt^3}{12} + 3(E_r - E_m)\alpha^2\beta\frac{bt^3}{12} + (E_r - E_m)\alpha^3\frac{bt^3}{12}\right] \quad (8)$$

where α indicates the part of effective reinforcement and β indicates the part of mortar, and are given as follows:

$$\Delta y = \alpha\frac{t}{2} \quad (9)$$

$$y_0 = (1-\alpha)\frac{t}{2} = \beta\frac{t}{2} \quad (10)$$

Eq. (8) can be written in terms of moment of inertia as follows:

$$M = a\{E_m + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_r - E_m)\}I_c \quad (11)$$

where I_c is the moment of inertia of the composite flexural section with respect to neutral axis which is given by:

$$I_c = \frac{bt^3}{12} \quad (12)$$

Table 1
Specification and properties of meshes.

Type of mesh	Wire diameter/ cross-section mm (in.)	Mesh openings mm (in.)	Modulus of elasticity (kN/mm^2)	Poisson's ratio
Square mesh	1.2 (0.047)	10.0 (0.39) × 10.0 (0.39)	138.0	0.3
Chicken mesh	0.8 (0.031)	11.2 (0.44) × 9.0 (0.35)	104.0	0.3
Geogrid mesh	3.0 (1.18) × 4.0 (1.57)	22.0 (0.86) × 22.0 (0.86)	75.0	0.4

1 N/mm^2 = 145 psi, 1 kN/mm^2 = 6.47 tsi.

Substituting the value of a from Eq. (11) into Eq. (5), yields:

$$\varepsilon = \frac{My}{\{E_m + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_r - E_m)\}I_c} \quad (13)$$

where

$$E_{com.} = E_m + (3\alpha\beta^2 + 3\alpha^2\beta + \alpha^3)(E_r - E_m) \quad (14)$$

Eq. (14) is the composite modulus of elasticity of the whole section which is finally expressed as follows:

$$E_{com.} = E_m + \{3R_r(1-R_r) + R_r^3\}(E_r - E_m) \quad (15)$$

where R_r is the effective reinforcement taken instead of α.

5. Materials and methods

Ordinary Portland cement (Type I) and river sand passing through JIS sieve No.2 (F.M. 2.33) were used for the preparation of mortar. The Young's modulus and Poisson's ratio of mesh wires and geogrids (G) obtained experimentally are depicted in Table 1 (JSCE) [10]. The diameters of wire, mesh openings and physical appearances of the meshes and dimensions used in this investigation are shown in Fig. 1. The tensile (yield) strengths of meshes were 450 MPa (65.25 ksi), 310 MPa (44.96 ksi), 380 MPa (55.11 ksi) and 250 MPa (36.25 ksi) for square, chicken, hybrid and geogrid meshes respectively.

5.1. Casting of test specimen

The specimens were cast in wooden moulds with open tops (Fig. 2). For all the specimens, water to cement ratio and cement to sand ratio was 0.5 by weight. Ordinary meshes obtained from the market were cut to obtain the desired size. The sand cement mortar layer was spread at the base of the mould. On this base layer, the first mesh was laid. The mesh layer was covered by another layer of mortar. The process was repeated until the specimen contains the desired number of mesh layers. Thus, the thickness of 30 mm (1.18 in.) was equally divided by the mesh layers, leaving a cover of 2.0 mm (0.07 in.) at the top and bottom surfaces. The type of mesh used and the number of layers of mesh were marked on the element. The meshes were uniformly arranged for all the specimens. The specimens were air-dried for 24 h for initial setting and then immersed in water for curing. After 28 days, the specimens were removed from water and air-dried for 48 h in room temperature of about 10 °C with relative humidity of about 40% before testing.

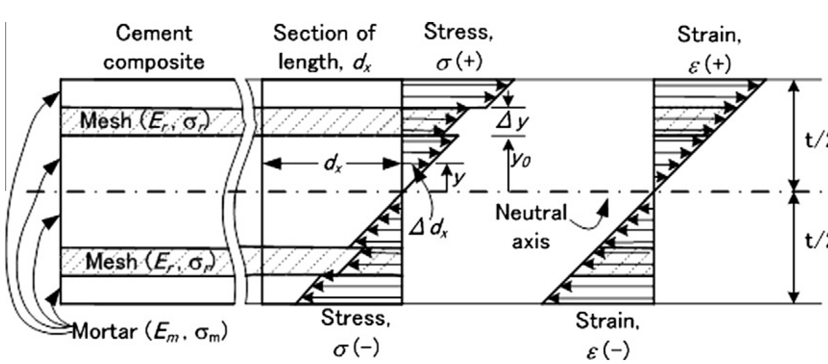

Fig. 3. Analysis of flexural cement composite section.

The size of the flexural specimens used in this research work was 200 × 400 × 30 mm (7.87 × 15.74 × 1.18 in.). Along with the flexural specimens, rectangular and cylindrical specimens were also cast in order to obtain the compressive strength of the cement composites. The compressive strength test results are shown in Table 2.

5.2. Testing of sample

All the elements were tested on a simply supported span of 360 mm (14.17 in.) under third-point loadings according to ASTM C78 [11]. The distance between the loading points is 120 mm (4.72 in.) with lever arms of 120 mm (4.72 in.) at both sides of the loading points. The load was applied in a vertically upward direction with the tension side up. A total number of 72 specimens (three specimens for each group) with number of mesh layers from 1 to 6 were tested. In this study, the deflections were measured at the mid-section of a simply supported beam as shown in Fig. 4. The equation for load–deflection relationship under third-point loadings is derived as:

$$\delta = \frac{PL^3}{56.35E_{com.}I_c} \quad (16)$$

where $E_{com.}$ is the composite modulus of elasticity of the whole section, I_c is the moment of inertia of the flexural section with respect to neutral axis, L is the span length, P is the applied load and δ is the deflection at mid-section. The modulus of elasticity of each specimen is, therefore, calculated experimentally by the following equation:

$$E_{com.} = \frac{PL^3}{56.35I_c\delta} \quad (17)$$

6. Results and discussion

The load–deflection relationships of cement composites with two layers of square, chicken, hybrid and geogrid meshes are shown in Fig. 5. The linear range of the load–deflection curves of cement composites for the above four types of meshes are shown in Figs. 6–9. The moduli of elasticity of cement composites for all the cases are calculated and given in Table 3. Similar to the form of theoretical equations (Eqs. (1) and (15)), the composite modulus of elasticity given in Table 3 can be expressed in the following form in terms of effective reinforcement.

$$E_{com.} = E_m + (\varphi R_r + \xi)(E_r - E_m) \quad (18)$$

Here, φ denotes mesh-type with values of 0.1265, 0.0858, 0.0327 and 0.0178 for the cement composites containing chicken mesh, hybrid mesh, square mesh and geogrid mesh, respectively.

Table 2
Properties of mortar specimens.

Specimens	Compressive strength (N/mm^2) (psi)	Modulus of elasticity (kN/mm^2) (tsi)	Poisson's ratio
Panels	28.26 (4097.70)	13.49 (87.28)	0.24
Cylinders	27.43 (3977.35)	17.45 (112.90)	0.25

Note: Average values of compressive strengths (27.84 N/mm^2) and moduli of elasticity (15.47 kN/mm^2) are used in Table 4.

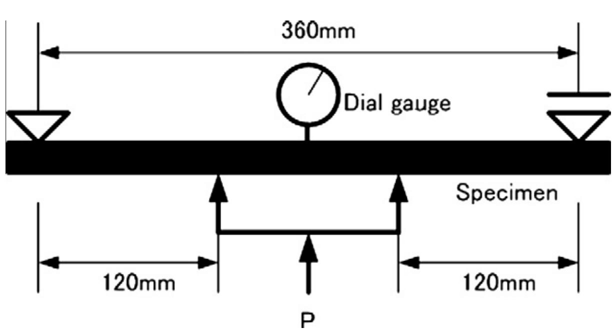

Fig. 4. Loading arrangement on a sample specimen.

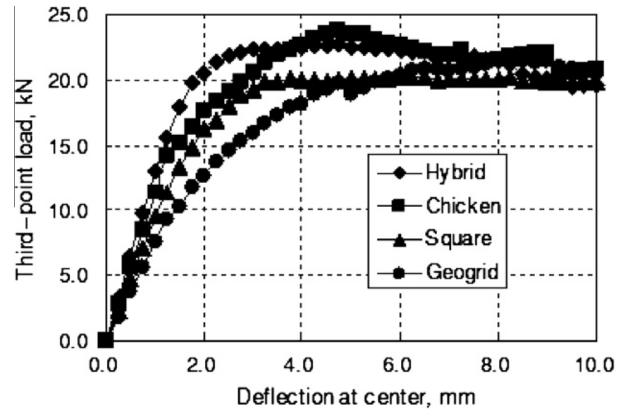

Fig. 5. Load-deflection relationships for calculation of $E_{com.}$ (kN/mm^2) with two layers of mesh.

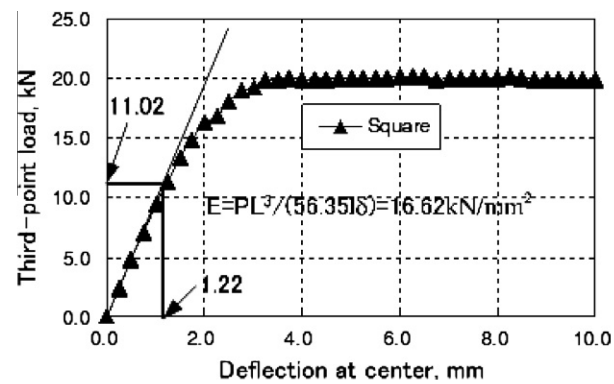

Fig. 6. Linear range of the load–deflection curve for square mesh (P = 11.02 kN, L = 360 mm, b = 200 mm, t = 30 mm, I = 45000 mm^4, δ = 1.22 mm).

The ζ indicates uncertainty regarding the little experimental data with values of 0.0258, 0.0326, −0.0089 and −0.0031 for the cement composites containing chicken mesh, hybrid mesh, square mesh and geogrid mesh, respectively. The value of ζ which appeared due to the uncertainty of little experimental data should be zero from the theoretical viewpoint. Thus, the composite modulus of elasticity from experimental data without uncertainty can be written as follows:

$$E_{com.} = E_m + \varphi R_r(E_r - E_m) \quad (19)$$

The moduli of elasticity calculated by the analytical and experimental equations are compared for cement composites reinforced with square mesh, chicken mesh, hybrid mesh and geogrid mesh

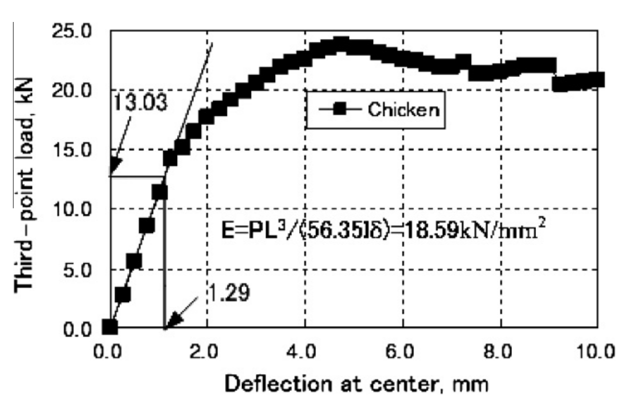

Fig. 7. Linear range of the load–deflection curve for chicken mesh (P = 15.55 kN, L = 360 mm, b = 200 mm, t = 30 mm, I = 45000 mm^4, δ = 1.41 mm).

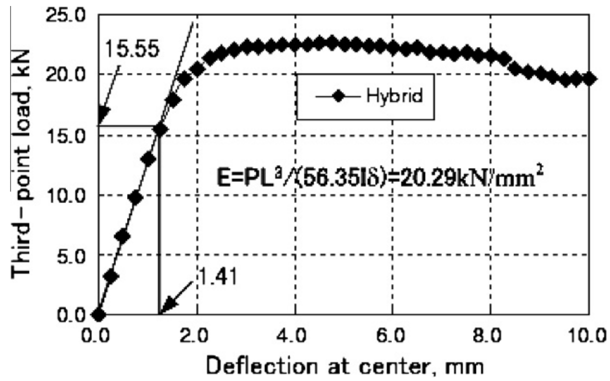

Fig. 8. Linear range of the load–deflection curve for hybrid mesh (P = 15.55 kN, L = 360 mm, b = 200 mm, t = 30 mm, I = 45000 mm^4, δ = 1.41 mm).

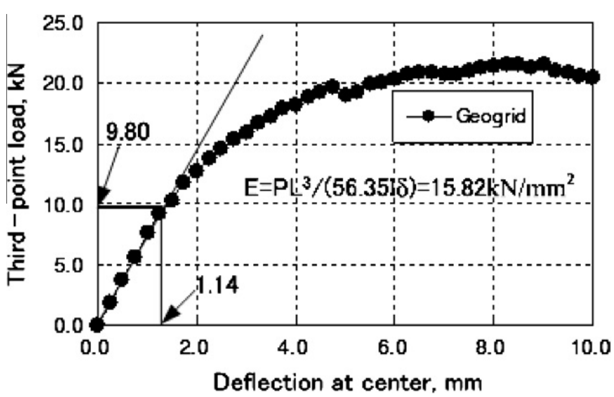

Fig. 9. Linear range of the load–deflection curve for geogrid mesh (P = 15.55 kN, L = 360 mm, b = 200 mm, t = 30 mm, I = 45000 mm^4, δ = 1.41 mm).

and are presented in Table 4. This is to note that Eq. (1) overestimates the moduli of elasticity of thin cement composites as compared to Eq. (15) (newly derived equation under flexural state) and Eq. (19) (equation obtained from experimental data) for all

Table 3
Modulus of elasticity of cement composite obtained by experiment.

Specimen	R_r (%)	E_{com}. (kN/mm^2) (average of three specimens)
Square mesh	0.38	15.75
	0.75	16.48
	1.13	16.85
	1.50	17.56
	1.69	18.75
	2.26	19.25
Chicken mesh	0.07	17.45
	0.14	18.47
	0.21	18.62
	0.28	18.73
	0.35	19.04
	0.42	20.85
Hybrid mesh	0.45	19.47
	0.52	20.13
	0.82	21.54
	0.89	21.94
	0.96	22.80
	1.34	23.98
Geogrid mesh	0.52	15.46
	1.03	15.76
	1.54	15.94
	2.06	16.65
	2.57	17.34
	3.08	18.25

1 N/mm^2 = 145 psi, 1 kN/mm^2 = 6.47 tsi.

Table 4
Comparison of modulus of elasticity (E_{com}.), kN/mm^2.

R_r (%)		Analytical		Experimental	
		Eq. (1)	Eq. (15)	Eq. (18)	Eq. (19)
Square mesh	0.50	16.08	15.58	15.48	15.49
	1.00	16.70	15.59	15.50	15.51
	1.50	17.31	15.61	15.52	15.53
	2.00	17.92	15.72	15.54	15.55
	2.50	18.53	16.01	15.56	15.57
Chicken mesh	0.50	15.91	15.55	15.55	15.53
	1.00	16.36	15.56	15.60	15.58
	1.50	16.80	15.57	15.66	15.64
	2.00	17.24	15.65	15.72	15.69
	2.50	17.68	15.86	15.77	15.75
Hybrid mesh	0.50	16.00	15.56	15.55	15.52
	1.00	16.53	15.58	15.59	15.56
	1.50	17.05	15.59	15.64	15.61
	2.00	17.58	15.68	15.69	15.65
	2.50	18.11	15.93	15.73	15.70
Geogrid mesh	0.50	15.77	15.52	15.47	15.48
	1.00	16.07	15.53	15.48	15.48
	1.50	16.36	15.54	15.48	15.49
	2.00	16.66	15.59	15.49	15.49
	2.50	16.96	15.73	15.49	15.50

1 N/mm^2 = 145 psi, 1 kN/mm^2 = 6.47 tsi.

the cases of cement composites containing square mesh, chicken mesh, hybrid mesh and geogrid mesh. This is obvious because the available equation was based on the law of mixture rule and the newly derived equation is based of flexural state. Interestingly the experimental results agree well with the results calculated by the newly developed equation.

The following opinions can be noted from the above compilation of the test results. When flexural modulus of elasticity is concerned, it is necessary to consider the factor $\left\{3R_r(1-R_r)+R_r^3\right\}$ of newly derived equation (Eq. (15)) or factor $\varphi R_r$of experimental equation (Eq. (19)) instead of using the available composite equation (Eq. (1)), directly (Fig. 10). If the difference of the modulus of elasticity of composite obtained by Eqs. (1) and (19) is defined by the term 'mesh factor (MF); this MF varies as 1.01, 0.73, 0.87 and 0.49 for cement composites containing square mesh, chicken mesh, hybrid mesh and geogrid mesh, respectively for a change of 1% reinforcement. Further, it has been found that these MFs need to be deducted from the composite modulus of elasticity calculated by the available analytical equation (Eq. (1)) in order to obtain a more reliable value of the flexural modulus of elasticity.

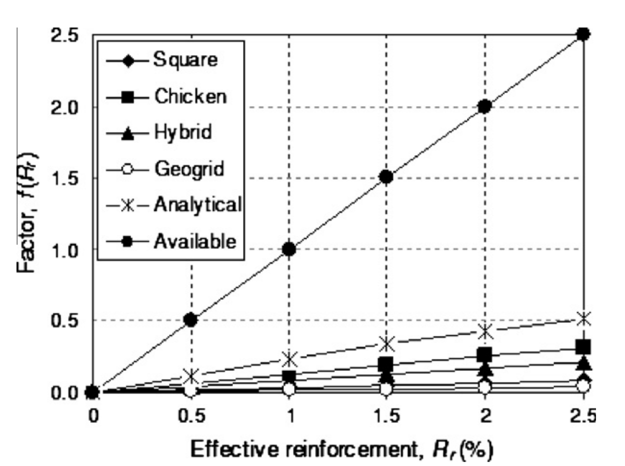

Fig. 10. Comparison of factor without uncertainties (factor, $\int(R_r)$ indicates R_r of Eq. (1), $\left\{3R_r(1-R_r)+R_r^3\right\}$ of Eq. (15) and φR_r of Eq. (19)).

7. Conclusions

In this paper, a flexural section of thin cement composite is theoretically analyzed and a simple design equation for flexural modulus of elasticity of thin cement composites reinforced with different types of meshes was derived. Laboratory experiments were conducted to check the validity of the new equation. The experimental results agree well with the results obtained by the proposed equations. This paper also recommends the necessity of mesh factor (MF) for the sake of ease in the design process when flexural composites are to be reinforced with different meshes.

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