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PSS-control as an ancillary service

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Abstract

This paper proposes that the control action provided by power system stabilizers (PSSs) to enhance system stability, be considered as one of the system ancillary services. To this effect, there is a need to formulate appropriate financial compensation mechanisms for the generators, in return for their service. At the same time, it is also important to identify which PSS is more crucial for system stability, and also those which could be even detrimental to overall system stability. A cooperative game theory-based approach using the Shapley value criterion is developed in this paper to identify the marginal contribution of each PSS to the total control effort. Accordingly, the method outlines appropriate allocation of payment to each generator involved in providing the PSS-control.

Keywords: Ancillary services; Cooperative game theory; Power system stabilizers; PSS-control service; Shapley value

1. Introduction

Power system stabilizers (PSS) have been widely accepted and recognized as important control devices that are essential for ensuring system stability, particularly the small signal stability phenomenon. They have been installed and have found application in practical large-sized power system [1]. A great deal of work has been reported in the literature pertaining to optimal tuning of PSS parameters using methods ranging from classical modal analysis and linear optimal control [2], adaptive and variable structure to more recent methods involving artificial intelligence techniques [3,4].

In deregulated electricity market environment, the problem of tuning and optimization of PSS parameters is a challenging issue that has not been addressed yet. Amongst the most important issues associated with PSS tuning in deregulated environment, the issue of responsibility and coordinated tuning of PSS are the most important ones. As of now, no definite guidelines have been established by Independent System Operators (ISO) or the equivalent authorities, with regard to this question. Nevertheless, some operating authorities have outlined certain rules on PSS installation requirements on synchronous generators. For example, Western Electricity Coordinating Council (WECC) requires that PSS be installed on every existing synchronous generator that is larger than 75 MVA or is larger than 30 MVA and is part of a generation complex that has an aggregate capacity larger than 75 MVA [5].

In deregulated power systems, the ISO is entrusted to ensure a required degree of quality, safety, reliability and stability and perform several other functions. *Ancillary services* are all those activities that are necessary to support power transmission, while maintaining reliable and stable operation and ensuring the required degree of quality and safety. These services thus include regulation of frequency and tie-line power flow, voltage and reactive power control, ensuring system stability, maintenance of generation and transmission reserves, and many others. According to NERC Operating Policy 10 [6], the following services are recognized as ancillary services:

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For maintaining generation and load balance:
- regulation service;
- load following service;
- contingency reserve service.

For bulk transmission system security:
- reactive power supply from generation sources;
- frequency response service.

For emergency preparedness:
- system black start capability.

1.1. PSS-control ancillary service

In an interconnected and deregulated power system with several generators equipped with PSS, the parameters of the PSS would have been optimally tuned in a coordinated manner by the ISO, or a similar entity. These PSSs render a service to the power system by way of providing stabilization action to small disturbances that occur in the system continually. The stabilization action is through auxiliary corrective signals to the reference of the automatic voltage regulator.

In the absence of this service, the system will most likely become unstable due to sustained low frequency oscillations. Evidently, this directly affects the transmission system security and reliability and hence the service provided by such PSS-control action can essentially be classified within the ancillary services definitions, as a service for bulk transmission system security.

The importance of the PSS to the system has been clearly understood and appreciated by power system engineers and operators. However, their worth and their contribution to system savings have never been investigated and outlined in an analytical manner. This is because of the complexity and difficulty associated with relating the system dynamic performance of a PSS to a "dollar figure", which would quantify savings accrued from a properly tuned PSS. Once the system savings to the ISO from a PSS (in dollar terms) are appropriately attributed, it would be possible to develop proper pricing mechanisms to compensate the generators for these ancillary services.

Consequently, the PSS-control service can qualify as an ancillary service and fall within the purview of ISO operating policies. The generators providing this service, and abiding by the ISO's instructions on optimal PSS parameter settings understandably, thus, are entitled to a payment for the service. Although it is extremely difficult to determine the cost incurred by a generator for providing the PSS-control service, nevertheless, under deregulation, a proper financial mechanism must exist to compensate for these costs [7].

Further, we should also recognize that each PSS would have different impact on the system in terms of its stabilization action. For example, PSS on Gen-1 could possibly be more vital to system stability than a PSS on Gen-3 and so on. This discriminatory behavior is highly dependent on the current operating context defined by the operating condition, type of event and the specific PSSs that are in service.

In the present work, we propose PSS-control as a service within the definitions of system ancillary services and develop appropriate mechanisms for financial compensation to synchronous generators for such services. We examine how system savings are accrued through PSS-control action and how to assign a quantitative "dollar figure" to the quality of system dynamics in the presence of PSS. Subsequently, we propose a cooperative game theoretic approach based on Shapley value concept to determine the marginal contribution of each PSS in the system, and hence how each PSS should be paid for the control service it provides. In other words, Shapley value is used to allocate payments to each player (i.e. generator equipped with PSS) in the system, depending on how important the PSS is to overall system stability.

It is important however to emphasize that this paper proposes a technique of sharing the cost savings from PSS and does not focus on parameter tuning aspects or controller designs.

1.2. The system investigated

Fig. 1 shows the well-known nine-bus, three-generator interconnected power system [8], which has been considered in this work for analysis. Loads are connected at buses 5, 6 and 8, respectively.

Although it is a fairly small-sized system, it has been very widely used by researchers for analytical studies, and is representative enough to demonstrate the proposed Shapley value-based scheme for determining the worth of PSS-control service.

For our analysis, the lead-lag PSS has been considered, with gain K_{C_i}, time constants T_{1_i} and T_{2_i}, and wash-out filter time constant T_{W_i}. The transfer-function representation of the PSS on the ith generator is given as follows:

$$u_i(s) = K_{C_i} \frac{sT_{W_i}}{1 + sT_{W_i}} \left(\frac{1 + sT_{1_i}}{1 + sT_{2_i}} \right)^2 \Delta\omega_i \quad (1)$$

In (1), u_i is the output signal from the PSS that provides the corrective action to damp the low frequency electro-

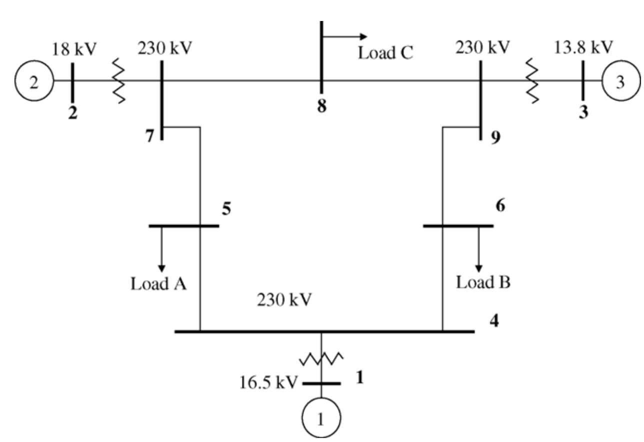

Fig. 1. Three-machine, nine-bus system.

mechanical oscillations. The lead-lag PSS derives its input from the rotor speed deviation, $\Delta\omega$.

2. Savings allocation and pricing of PSS-control service

In *game theory* parlance, in accordance to the way in which the players interact with one another in a given game and the extent to which they influence each other's decisions, the game can be classified into two major categories, namely *cooperative* or a *noncooperative game*.

In a *noncooperative game*, strategies are chosen by the players independently, the rules would not allow players to join forces and coordinate actions for better outcomes. On the other hand, in a *cooperative game*, the players have strictly identical interests or certain agreements/other commitments are enforceable on the players [9].

As we have mentioned earlier, in most deregulated power systems, it would be the responsibility of the ISO or a similar entity to evolve a coordinated PSS tuning and operation strategy based on certain system-wide objective function.

Therefore, the PSSs operation can be modeled as a cooperative game, having as its characteristic function a savings formulation, which is based on the objective function used in the tuning process, and where one or more generators are considered together (in all possible *coalitions*) to obtain the fair revenue allocation.

Thus, our analysis of the worth of the PSS-control ancillary service should be based on two important issues: (a) how the coalitions are formed amongst the PSS and consequently (b) how the benefit from a PSS service is allocated. In these two interrelated issues, our main concern is to obtain the most likely outcome from various game situations. Especially, when it comes to distribution of the savings due to PSS-control action, the revenue corresponding to a PSS in a particular coalition is very difficult to evaluate.

An intuitively attractive solution concept for n-person cooperative games with transferable utility (payoff, in our case) has been proposed by Shapley in 1953 and is called the Shapley value. The Shapley value can be defined by means of the following postulates [9]:

(a) *Joint efficiency*—The sum of all players' payoffs equals the value of the grand coalition (n player coalition, which is the highest joint payoff the n players can achieve within the game).

To explain this, the sum of the individual payments to each PSS is what the ISO accrues as benefit or savings by having PSS at all generators (grand coalition).

(b) *Zero payoff to any dummy player*—If a player fails to contribute anything to the value of any coalition that he may join, then he is called a *dummy player*. The payoff to a dummy player is zero.

In a multi-machine power system a generator that is completely isolated from the interconnected power system and from other generators can be called a *dummy player* because it has no role in providing for system stabilization service. Hence, it is not eligible to receive any payment from the ISO.

(c) *Symmetry*—If all players are identical, they share the total system savings equally.

This postulate however does not play a significant role in our analyses since all our generators have different characteristics.

(d) *Additivity*—The payoff of any given player is equal to the sum of all payoffs it would receive as a member of all possible coalitions.

This means that the payment actually received by a generator, reflects how each PSS contributes to enhancing the system stability, since it is the sum of its contribution in all possible contributions.

The marginal contribution Ψ of a player i in coalition C ($\forall\, i \in C$) will be given as shown in [10] by:

$$\Psi_i(C) = v(C) - v\,(C/\{i\}) \tag{2}$$

where $v(C)$ is the payoff (savings) resulted from coalition C.

The Shapley value ϕ, which is the weighted average of the marginal contributions of a player i in all possible coalitions, is hence given by:

$$\phi_i = \sum_{C} C_w(C) \cdot \Psi_i(C) \tag{3}$$

where $C_w(C) = \frac{(q-1)!(n-q)!}{n!}$, q is the size of a coalition.

Having obtained the contribution of a PSS to system savings, a mechanism for financial compensation to synchronous generators for their PSS-control service can now be devised for the ISO. The payment ρ we propose here has the following structure:

$$\rho_i = a_i + \phi_i \tag{4}$$

In (4), a represents the component of payment associated with availability of the PSS at a generator. This component is payable to the generator for having the PSS installed and adhering to ISO's instructions on parameter settings. The generator is entitled to this component of payment even if the ISO instructs that the said PSS remain off-line.

The second component of ρ given in (4) denotes the variable payment component as given in (3), proportional to the "worth" of the PSS in system stabilization, and is determined using the proposed Shapley value-based method.

An important aspect in ancillary services is the way in which they are handled and managed by the ISO. Many times, ancillary services, such as spinning reserve, regulation, etc., are part of the market clearing process and the suppliers have to simultaneously bid for energy and these ancillary services [11]. On the other hand, certain services such as reactive power support can be on long-term contracts, as in UK where bi-annual tenders are held to establish the contracts [12]. It is envisaged that PSS-control ancillary service would also be on long-term contracts between generators and ISO, so that short-term price volatility due to emergency system conditions do not significantly affect the payment structure.

3. Results and analysis

3.1. *Optimal PSS, minimal PSS, goodness index and savings*

3.1.1. *Optimal PSS design*

As mentioned in Section 1, a great deal of work has been reported on optimal tuning of PSS parameters. This paper shall not dwell upon PSS tuning methods and related issues. Instead, for the purpose of the present study, we shall use previously reported results by the authors, where a Lyapunov equation-based criterion was implemented within a genetic algorithm framework to obtain the optimal set of PSS parameters [13].

A quadratic performance index *J*, which measures the performance of the PSS for a set of parameters and operating conditions, is defined in (5):

$$J = \int_0^{\infty} (\underset{\sim}{x}^{\mathrm{T}} Q \underset{\sim}{x})\,\mathrm{d}t \tag{5}$$

In (5), $\underset{\sim}{x}$ is the state vector while **Q**, the weighting matrix, is positive semi-definite and denotes the importance attached to the different state variables in the optimization process. The value of *J* can be numerically computed using (6), where **P** is a positive-definite matrix obtained by solving the Lyapunov equation given in (7).

$$J = \underset{\sim}{x}^{\mathrm{T}}(0) \cdot P \cdot \underset{\sim}{x}(0) \tag{6}$$

$$A^{\mathrm{T}} \cdot P + P \cdot A = -Q \tag{7}$$

The PSS parameters thus obtained for a perturbation of 0.01 per unit at Gen-1 as given in [13] are shown in Table 1.

3.1.2. *Minimal PSS*

Since the system without PSS is not stable when subjected to a small perturbation, a minimal PSS (MPSS) is first determined. We shall refer to MPSS being that set of PSS parameters for which, with a minimum control action from PSS, the system is stable. Evidently, the MPSS is a sub-optimal PSS and hence any optimal PSS would provide a better performance. The performance index with the MPSS is termed as the *reference performance index* (RPI). The parameters of the MPSS together with its corresponding performance index (RPI) are also presented in Table 1.

Table 1
Minimal PSS and optimal PSS in the grand coalition

	PSS	
	MPSS	Optimal PSS
Gain (p.u.)		
G-1	41.8	45.06
G-2	–	45.52
G-3	1	2.13
Time constant (s)		
G-1	0.1	0.17
G-2	–	0.06
G-3	0.01	0.44
Performance index, *J* (p.u.)	$2{,}941.2 \times 10^{-6}$	3.1524×10^{-6}

3.1.3. *Goodness index*

We now define a new index, referred to as the *goodness index* (GI), which is a measure of the rotor angle oscillation damping and settling time of the perturbed system, with an optimally tuned PSS, as compared to MPSS. This is defined as follows:

$$\mathrm{GI} = \mathrm{RPI} - J \tag{8}$$

Our objective is to evaluate the contribution of a PSS to system stability and its worth in bringing about cost savings. We determine all possible coalitions in which a PSS may participate and hence obtain the set of optimal PSS parameters for each coalition, using the GA-based PSS tuning method reported in [13]. Table 2 shows the various coalitions which can be formed by the machines — all equipped with PSS — of the system considered for analysis. Also shown are the corresponding optimal PSS parameters, and the performance (PI) and goodness indices (GI). It is to be noted that for coalitions (2), (3) and (2–3), the system is not stable and hence these coalitions are not feasible.

- Coalition (1–3) has a low GI and both the gain K_{C} and time constant T_1 of PSS-3 tend to their lower bounds (which are set in the genetic search algorithm at 1.0 p.u. and 0.01 s,

Table 2
Coalitions, optimal PSS and goodness index

Coalition, *C*	Optimal PSS		$\mathrm{PI} \times 10^{-6}$	GI
	Gain ($K_{\mathrm{C}_1}, K_{\mathrm{C}_2}, K_{\mathrm{C}_3}$)	Time constant ($T_{1_1}, T_{1_2}, T_{1_3}$)		
1	(62.658, –, –)	(0.1215, –, –)	6.3079	2934.89
2	Infeasible	Infeasible	2941.2	0
3	Infeasible	Infeasible	2941.2	0
1–2	(43.167, 47.513, –)	(0.176, 0.098, –)	3.4068	2937.79
1–3	(65.74, –, 1.0)	(0.1192, –, 0.01)	6.3468	2934.85
2–3	Infeasible	Infeasible	2941.2	0
1–2–3	(45.06, 45.52, 2.13)	(0.17, 0.06, 0.44)	3.1524	2938.05

respectively). Thus, we can say that the coalition (1–3) converges to coalition (1), PSS-3 being naturally minimized through the optimization process.

- However, in spite of that, PSS-3 is a very important player in coalition (1–2–3), as reflected by the corresponding performance indices.

3.1.4. Savings to system

In this sub-section, we attempt to correlate the dynamic performance index of the PSS to an economic index, in dollar terms, that represents the benefit to the system by having the PSS. We proceed as follows:

(a) Let us consider the system without PSS at any machine. In such a case, the system is unstable and is in a blackout condition. This understandably has immense cost on the system, which is though very difficult to ascertain, and is beyond the scope of discussion of this paper. For our system considered, the total load of 315 MW remains unserved when there is no PSS and the system is unstable.

(b) Now let us consider the MPSS which is a sub-optimal PSS and barely stabilizes the system, having a rather high performance index ($J = 2941.2 \times 10^{-6}$, see Table 1). However, by virtue of this PSS the system is able to serve all the customer loads. Hence, it brings about savings to the system (referred to as *base savings*, S_{base}), as compared to (a). Assuming the *cost of unserved energy* of US$ 100/MWh, S_{base} from MPSS equals US$ 31,500.

(c) Next, let us consider an optimally tuned PSS—the grand coalition (1–2–3). The system performance index (PI) is now significantly improved ($J = 3.1524 \times 10^{-6}$, see Table 1), because of reduced oscillations and power swings in the system. This brings about further savings to the system as compared to base savings, and shall be referred to as *incremental savings* (ΔS) from grand coalition.

(d) A *savings rate* (SR) can now be defined as the savings brought about by the MPSS, per unit of its PI (i.e. RPI), as given in (9):

$$\text{SR} = \frac{S_{\text{base}}}{\text{RPI}} \tag{9}$$

Without any loss of generality, we can assume that SR remains constant over all coalitions and hence can be used to determine the incremental savings and the total savings. For the system considered, SR is obtained as follows:

$$\text{SR} = \frac{31,500}{2,941.2 \times 10^{-6}} = 10.7099 \times 10^{6}\ (\text{US\$/p.u. PI})$$

(e) Finally, the incremental savings from a coalition (of optimal PSSs) can be determined as follows:

$$\Delta S_{\text{C}} = \text{SR} \cdot \text{GI}_{\text{C}} \tag{10}$$

Table 3
Calculation of savings from PSS operation

Coalition	GI (p.u.)	Incremental savings, ΔS (US$)	Total savings, S (US$)
No PSS	Infeasible	–	–
MPSS	0	31,500.00	31,500.00
1	2,934.89	26,024.00	62,932.44
1–2	2,937.79	27,771.38	62,963.51
1–3	2,934.85	25,979.38	62,932.03
1–2–3	2,938.05	27,933.84	62,966.24

(f) Thus, the total savings, S_{C}, from a coalition can be obtained as follows:

$$S_{\text{C}} = S_{\text{base}} + \Delta S_{\text{C}} \tag{11}$$

For the system considered, the incremental and total savings corresponding to each coalition are given in Table 3. Observe that the system achieves the highest savings with the grand coalition (i.e. coalition 1–2–3), which is worth US$ 62,966.24. This is the savings incurred by the ISO from having the PSS installed, optimally tuned and operating at all generators.

3.2. Determination of Shapley values

As we have argued earlier, the individual generators would be entitled to a payment in return for providing the PSS-control ancillary service. Consequently the ISO's problem is to allocate the savings achieved from PSS operation (US$ 62,966.24 in our example) in a fair and rational manner.

We have discussed earlier in Section 2, the theoretical background of cooperative game theory and assessment of contribution of individual players in a game. The method to calculate Shapley values for each player in the game was also outlined. Using the approach described therein, we obtain the Shapley values for each generator PSS of our example system, as described in Table 4.

From Table 4, we observe that PSS-1 receives the highest payoff (US$ 62,948.82), PSS-2 receives US$ 16.52, while PSS-3 receives the least payoff of US$ 0.84. We can explain the highest revenue allocation to PSS-1 as follows:

- From Table 2, it is evident that all feasible coalitions always include PSS-1. Moreover, none of the coalitions that do not include PSS-1 is stable.
- The GI of all coalitions does not differ significantly as compared to GI of coalition (1) (Table 2).

Therefore, we can conclude that PSS-1 has a dominant effect in terms of providing PSS-control ancillary service and is hence entitled to such a large payment. Similarly, we observe that PSS-2 and PSS-3 are contributing less to system stabilization service and therefore receive payments that are in proportion to their role in providing PSS-control service. Moreover, PSS-3 even appears to have a detrimental effect to system stability if it would be involved in coalition (1–3).

Table 4
Shapley value calculation

C_i	1	2	3	1–2	1–3	2–3	1–2–3	SV
GI	2,934.89	0	0	2,937.79	2,934.85	0	2,938.05	
W	1/3	1/3	1/3	1/6	1/6	1/6	1/3	
S	62,932.44	0	0	62,963.51	62,932.03	0	62,966.24	
PSS-1								
MC	62,932.44	–	–	62,963.51	62,932.03	–	62,966.24	62,948.82
SVC	20,977.48	–	–	10,493.92	10,488.67	–	20,988.74	
PSS-2								
MC	–	0	–	31.07	–	0	34.211	16.58
SVC	–	0	–	5.18	–	0	11.404	
PSS-3								
MC	–	–	0	–	−0.417	0	2.725	0.84
SVC	–	–	0	–	−0.0694	0	0.9082	
System savings, *S*								62,966.24

It is also to be noted in Table 4 that the sum of the total payoffs made by the ISO is the total savings accrued by it (as determined in Section 3.1.4) from optimal PSS operation.

3.3. Effect of bias induced by perturbation

Linear analysis techniques have been commonly used in order to obtain linear system models suitable for small signal stability analysis and PSS tuning. The linearized models of power systems are obtained by small perturbation analysis. Note that in our example too, the system was simulated by applying a 0.01 per unit perturbation in the mechanical torque of Gen-1.

From the evaluation of Shapley values in Section 3.2, it was evident that PSS-1 is dominant in providing system stabilization service. This leads us to question whether this is an inherent characteristic of the system, that it requires the maximum PSS-control action from Gen-1, or a feature that has been carried through by the way the system is perturbed and hence the tuning methodology.

To examine the above issue, we now consider a case where all generator mechanical torques are perturbed by 0.01 per unit, simultaneously. Subsequently, we re-tune the PSS parameters for the new perturbation scenario using the same GA-based approach described in [13]. The new set of optimal PSS parameters corresponding to all possible coalitions, and the associated PI and GI values are given in Table 5.

From Table 5, the following observations are made:

- Comparing PSS parameters in the grand coalition (1–2–3) obtained by perturbing Gen-1 only (Table 2), with those obtained by simultaneously perturbing all generators (Table 5), we observe that these are now more evenly weighed.
- The observed detrimental behavior of PSS-3 is even more pronounced in this case.

Table 6 shows the calculations of Shapley values for the case of simultaneous perturbation of all generators.

From Table 6, we observe the following:

- As in the previous case (shown in Table 4), PSS-1 retains its dominant character and receives the highest payment (US$ 62,601.96), while PSS-2 now receives a significantly higher amount (US$ 713.13).
- PSS-3 has a negative Shapley value, that is, it would receive a negative component of payment for introducing an overall detrimental effect on system stability. This implies that in the payment function (3), ϕ_i is negative, and the generator's overall payment is negatively affected.
- This is evident from the fact that PSS-3 has a negative marginal contribution in coalition (1–3). Although the

Table 5
Coalitions, optimal PSS and goodness index—all generators perturbed case

Coalition, *C*	Optimal PSS		PI × 10^{-6}	GI
	Gain (K_{C_1}, K_{C_2}, K_{C_3})	Time constant (T_{1_1}, T_{1_2}, T_{1_3})		
1	(70.784, –, –)	(0.1092, –, –)	12.291	2,928.91
2	Infeasible	Infeasible	2,941.2	0
3	Infeasible	Infeasible	2,941.2	0
1–2	(71.15, 48.7, –)	(0.1602, 0.1275, –)	1.924	2,939.28
1–3	(18.93, –, 1.0)	(0.145, –, 0.407)	195.210	2,745.99
2–3	Infeasible	Infeasible	2,941.2	0
1–2–3	(64.93, 59.88, 40.8)	(0.28, 0.195, 0.214)	0.636	2,940.56

Table 6
Shapley value calculations—all generators perturbed case

C_i	1	2	3	1–2	1–3	2–3	1–2–3	SV
GI	2,928.91	0	0	2,939.28	2,745.99	0	2,940.56	
W	1/3	1/3	1/3	1/6	1/6	1/6	1/3	
S	62,868.36	0	0	62,979.39	60,909.32	0	62,993.19	
PSS-1								
MC	62,868.36	–	–	62,979.39	60,909.32	–	62,993.19	62,601.96
SVC	20,956.12	–	–	10,496.56	10,151.55	–	20,997.73	
PSS-2								
MC	–	0	–	111.028	–	0	2,083.87	713.13
SVC	–	0	–	18.505	–	0	694.62	
PSS-3								
MC	–	–	0	–	−1,959.05	0	13.795	−321.91
SVC	–	–	0	–	−326.508	0	4.598	
System savings, *S*								62,993.19

grand coalition (1–2–3) returns with the highest system savings, if say, due to a contingency, PSS-2 is out of service, the coalition (1–3) will provide a highly inferior performance even as compared to coalition (1).
- It should also be observed that the figure of total savings made by the ISO in this case (US$ 62,993.19) is slightly higher than that made in the previous case (US$ 62,966.24), and this can only be explained as dynamic interactions between machines during the small signal stability event.

3.4. ISO imposed nonoperating constraint on a PSS

As observed from Tables 4 and 6, PSS-3 has little or negative contribution, respectively, to overall system stability. Therefore, it is reasonable to investigate the system behavior without a PSS at Gen-3. Table 7 shows in a similar manner as before (in Tables 4 and 6), the Shapley value calculations for the system with only PSS-1 and PSS-2, for (a) a small perturbation of 0.01 per unit at Gen-1 only (referred to as case C-1) and (b) small perturbation of 0.01 per unit at all generators (referred to as case C-2). From Table 7, the following inferences can be drawn:

- The system savings of US$ 62,979.4 is now distributed between PSS-1 and PSS-2; we note that PSS-1 receives a major share of the savings.
- The system savings do not change considerably when PSS-3 is removed from the system (for both cases). This will also be verified later by comparing the system dynamic performance of coalitions (1–2) and (1–2–3).
- Hence, PSS-3 can be removed from the system, without a significant compromise on system savings. Moreover, we eliminate the risk of having PSS-3 involved in certain coalition (such as 1–3), in which it exhibits an overall detrimental effect on system stability.

Figs. 2 and 3 show the angular speed variations recorded at the rotor of Gen-1 in both cases considered (C-1 and C-2), for coalitions (1–2) and (1–2–3), respectively.

As indicated by the performance indices as well, the system exhibits a fairly well damped dynamic oscillation when system is operated with all PSSs in service and with PSS-3

Table 7
Shapley value calculations—no PSS at Gen-3 case

C_i	1		2		1–2		SV	
	C-1	C-2	C-1	C-2	C-1	C-2	C-1	C-2
GI	2,934.89	2,928.91	0	0	2,937.79	2,939.28		
W	1/2	1/2	1/2	1/2	1/2	1/2		
S	62,932.44	62,868.36	0	0	62,963.51	62,979.39		
PSS-1								
MC	62,932.44	62,868.36	–	–	62,963.51	62,979.39	62,947.9	62,923.9
SVC	31,466.22	31,434.18	–	–	31,434.18	31,489.69		
PSS-2								
MC	–	–	0	0	31.07	111.03	15.535	55.514
SVC	–	–	0	0	15.535	55.514		
System savings, *S*							62,963.5	62,979.4

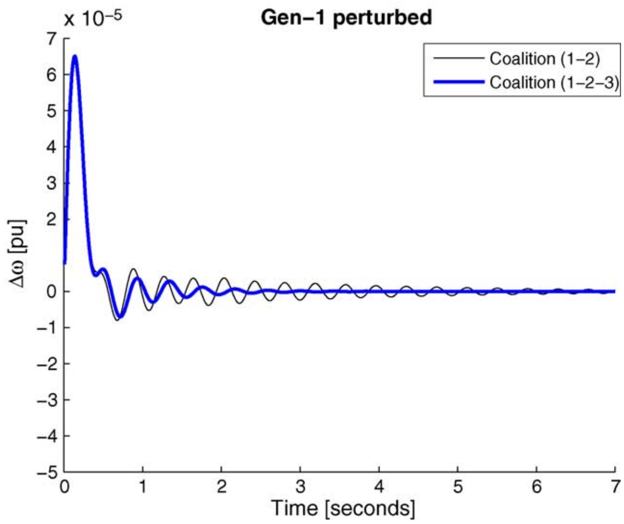

Fig. 2. Gen-1 rotor oscillations when Gen-1 perturbed, for coalitions (1–2) and (1–2–3).

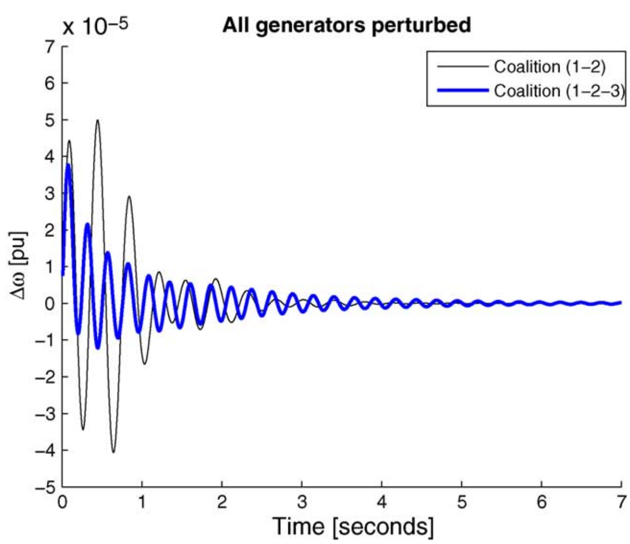

Fig. 3. Gen-1 rotor oscillations when all generators are perturbed simultaneously, for coalitions (1–2) and (1–2–3).

out of service, as well. Therefore, since the system can be operated with rather similar performance with or without PSS-3 in service, and also considering the risk posed by PSS-3 in certain operating situations, the ISO can constrain Gen-3 to operate with its PSS offline.

3.5. Note on computational aspects

Despite its benefits, Shapley value is not a common allocation method. One of the reasons is that for large systems, the computational costs can be significant, considering the fact that in a system with n generators equipped with PSS, the possible number of coalitions would be $2^n - 1$.

It is therefore important to discuss how this scheme can be applicable to realistic power systems. At this stage, we should mention that lot of work needs to be done in this area to further establish this concept so that it is applicable in practical systems. Nevertheless, we attempt to discuss a few ways in which the computational burden could be reduced:

- *Optimal siting of PSS*—As reported in literature, it is not likely that all generators have to be equipped with PSS, and hence the grand coalition should take into account the optimal siting decision, so that the number of PSS actually in operation is far less than the number of generators, n. This constitutes an inherent and considerable reduction in the number of coalitions to be analyzed.
- *Dynamic equivalencing of large power systems*—By either eigenvalue-, coherency-based or estimated dynamic equivalents, the system is reduced to a manageable-sized power system. Thus, dynamic equivalents for generators within a given area would be used, rather than isolated generating units. For example, the three generators of this paper can be thought of as to be representing a three-area system. Such dynamic equivalence of generators located within very close electrical distances is possible, and consequently the computational size can be significantly reduced.

4. Concluding remarks

In this paper, it is argued that power system stabilizer control actions be regarded as a system ancillary service towards bulk transmission system security. To this effect, a novel scheme for determining individual contribution of a PSS to the overall system stability is proposed. Subsequently, we formulate a method for rational allocation of payoffs to generators for their PSS-control service. The payoff allocation is based on cooperative game theory, using the concept of Shapley values. The system savings accrued from a grand coalition of PSSs providing control service are allocated in a fair and rational manner, using our proposed approach, which is based on weighted marginal contribution of a PSS in all coalitions it may be part of, thus reflecting better the role and importance of that PSS to the system.

For the example system considered, it was demonstrated how the total system savings from the grand coalition are allocated to the three PSSs. It was observed that PSS-1 received the highest payoff, thereby signifying its importance to system stability. On the other hand, PSS-3 was detrimental to system savings as well as system stability in certain coalitions. Hence, it received the least payoff, or even a negative payoff.

A further step in our investigation was to examine the system behavior without PSS-3, and it revealed that neither system savings nor system stability was considerably affected. Therefore, PSS-3 could be kept out of service, especially since in certain coalitions it turns out to have a detrimental effect on the systems stability and savings.

Appendix A

List of symbols

$\mathbf{A}$	state matrix
C	set of coalitions
GI	goodness index
i	index for players in the game
J	dynamic performance index
K_C	PSS gain
MC	marginal contribution
MPSS	minimum PSS
n	total number of players
N	set of all n players
PI	performance index
PSS	power system stabilizer
q	size of a coalition
$\mathbf{Q}$	weighing matrix
RPI	reference performance index
S	savings
SR	savings rate
SVC	Shapely value component
SV	Shapely value
ΔS	incremental savings
T_1, T_2	PSS time constants
T_W	PSS wash-out filter time constant
u_i	control signal from the ith PSS
$v(C)$	payoff from a coalition (or savings)
W	weight on a coalition
$\underset{\sim}{x}$	state vector
ϕ_i	weighted average of the marginal contributions of player i in all possible coalitions (Shapley value)
Ψ_i	marginal contribution of player i in a coalition

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