Available online at www.sciencedirect.com
ScienceDirect
Composite Structures 79 (2007) 125–132
COMPOSITE STRUCTURES
www.elsevier.com/locate/compstruct

Stress analysis of non-conventional composite pipes

M.A. Wahab [a,b,*], M.S. Alam [a], S.S. Pang [a], J.A. Peck [a], R.A. Jones [b]

[a] *Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA*
[b] *Commercial and Marine Products, EDO Fiber Science, Salt Lake City, Utah, UT 84116, USA*

Available online 10 January 2006

Abstract

In this study stress and failure analyses of polygonal-shaped composite pipes have been performed; five different shapes of circular, triangular, rectangular, pentagonal and hexagonal pipes are considered. The variation of peak hoop stress over different corner fillet radii, fiber orientation under internal pressure, and torsion loading have been investigated. Failure analysis methods utilized maximum principal stress and Tsai–Wu criteria.

Keywords: Non-conventional composite pipe; Rectangular; Semi-circular; Triangular; Pentagonal and hexagonal cross-sectional pipes; Finite element method; Failure analysis; Torsion

1. Introduction

In this study pipe with the following cross-sectional profiles (Fig. 1) have been analyzed: (1) circular; (2) rectangular; (3) triangular; (4) pentagonal; and (5) hexagonal.

The reason for this particular selection of cross-sectional shapes is to increase the scope of applicability for piping solutions when space is limited. Special consideration has been given to pipe bundles, i.e. multiple pipes that can be clustered to fit into a standard square or circular profile (Fig. 2). In this manner four triangle-shaped pipes or two rectangle-shaped pipes can be clustered into a square profile and two semi-circular pipes can be clustered into a circular profile. This allows for multi-use piping in a commercial sector where versatility translates into cost savings.

The principal obstacle to filament-winding pipes of unconventional cross-sectional shape is the impracticality of winding large flat faces. Convex structures are ideal for winding because the tension and winding angle of the fiber is kept relatively constant by the uniformly convex shape of the mandrel onto which the fibers are drawn. In order to institute the same level of confidence in winding pipes with flat faces the kinematics for rotating bodies which possess the profiles seen in Fig. 1 must be evaluated and the filament-winding process must be mechanically or computationally controlled to overcome the hindrances inherent with winding flat-faced structures.

2. Theoretical analysis

2.1. Circular pipe under internal pressure

The functional design of a composite pipe is based on the working fluid, temperature, and internal pressure. The design factors, which those conditions influence, are the resin (matrix), reinforcing material (fiber), and wall thickness of the pipe. The functional design of a composite pipe is well established for a pipe having a circular or conventional cross-sectional profile. The stress–strain distribution is easily derived from the simple geometry and applicable material properties; optimization is possible from this information.

The stresses in a thin-walled composite pipe (Fig. 3) can be derived if the internal pressure is known [1].

* Corresponding author. Address: Department of Mechanical Engineering, Louisiana State University, Baton Rouge, LA 70803, USA. Tel.: +1 225 578 5823; fax: +1 225 578 5924.
E-mail address: wahab@me.lsu.edu (M.A. Wahab).

0263-8223/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2005.11.054

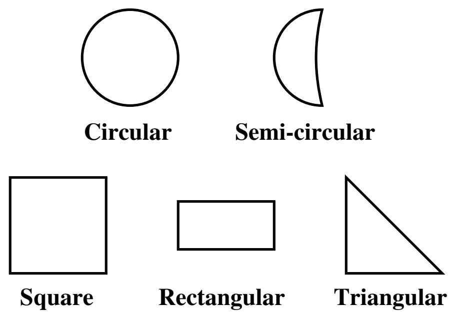

Fig. 1. Proposed cross-sectional profiles.

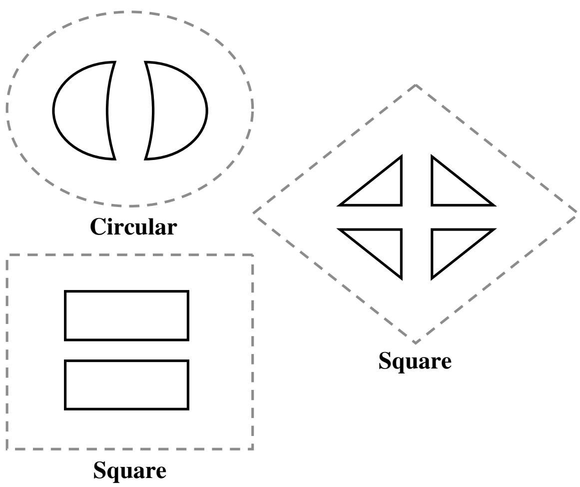

Fig. 2. Proposed pipe bundling arrangements.

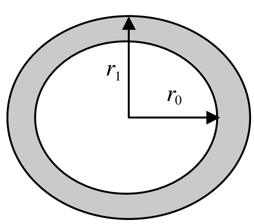

Fig. 3. Convention for analytic solution of conventional pipe.

$$\begin{aligned}\sigma_{r=r_1} &= 0\\ \sigma_{\theta=\theta_1} &= \frac{r_0 p}{(r_1 - r_0)}\\ \sigma_{z=z_1} &= \frac{r_0 p}{2(r_1 - r_0)}\end{aligned} \tag{1}$$

Of course, these equations are well established for conventional pipes made with isotropic materials (i.e., material having two-independent constants); but the pipes in this study are anisotropic (i.e., material having 21-independent constants) and both conventional and unconventional.

For circular-shaped pipes, the optimum winding pattern can be found using the following equations:

$$\begin{aligned}f &= \frac{rP}{t}\sin^2\theta\\ f_1 &= f\sin^2\theta\\ f_2 &= f\cos^2\theta\end{aligned} \tag{2}$$

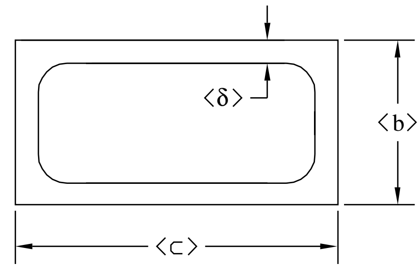

Fig. 4. Torsion in a thin pipe.

where f is the wall stress in the wound direction, f_1 is the hoop (circumferential) stress, f_2 is the longitudinal (axial) stress, r is the mean radius of the pipe, t is the laminate thickness, P is the burst internal pressure, and θ is the winding angle.

The optimum winding angle for a circular pipe can be deduced from the equations in (2) as follows:

$$f_1 = 2f_2 \tag{3}$$

Therefore, for internal pressure loading only, $\theta = 54.74°$ (all angles are taken with respect to the longitudinal axis of the pipe/mandrel). An alternative angle or combinations of angles in different layers will optimize the design for variant loading conditions.

2.2. Torsion in rectangular pipe

An approximate solution of the torsional problem for thin pipes can easily be obtained by using the membrane analogy [2]. The following formula for calculating shearing stresses in a rectangular pipe as shown in Fig. 4 was derived by Bredt [2],

$$\tau = M_t/2A\delta \tag{4}$$

where τ is torsion shear stress, M_t is torque, A is cross-sectional area and δ is thickness.

2.3. Numerical analysis

Finite element method (FEM) is used to evaluate the performance of the developed non-conventional pipes under pressure loading and torsion loading. The finite element models of those geometries (circular, triangular, rectangular, pentagonal and hexagonal) have been created and analyzed by authors' written ANSYS input files. Thickness and length of each model are 6.35 mm and 152.4 mm, respectively. The periphery at the mid-thickness of all models is 479 mm. Both ends of the models are fixed in all degrees of freedom. The distributed internal applied pressure for all cases is 689.5 kPa. The material properties are as follows: $E_{\mathrm{hoop}} = 15.2$ GPa, $E_{\mathrm{axial}} = 9.7$ GPa, $G_{xy} = 9.7$ GPa, Poisson ratio's = 0.4, density = 1.7 g/cm^3. The finite element model of a triangular shape pipe with boundary condition is shown in Fig. 5 and the applied internal pressure is shown in Fig. 6. The fiber orientation at different layers is shown in Fig. 7. To find a suitable element

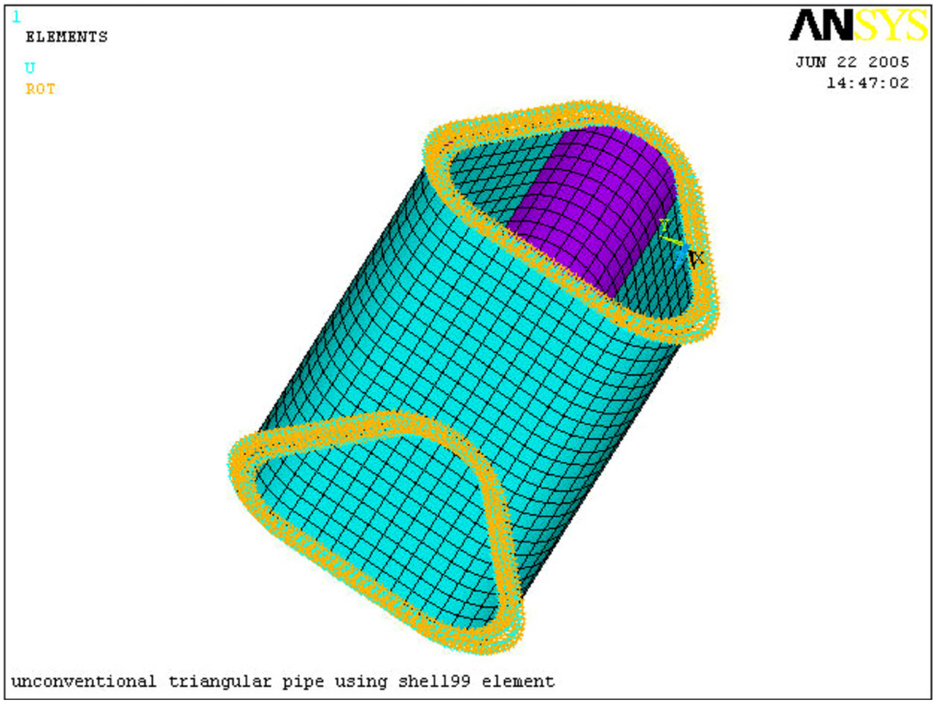

Fig. 5. Finite element model of triangular shape pipe showing both end fixed boundary conditions.

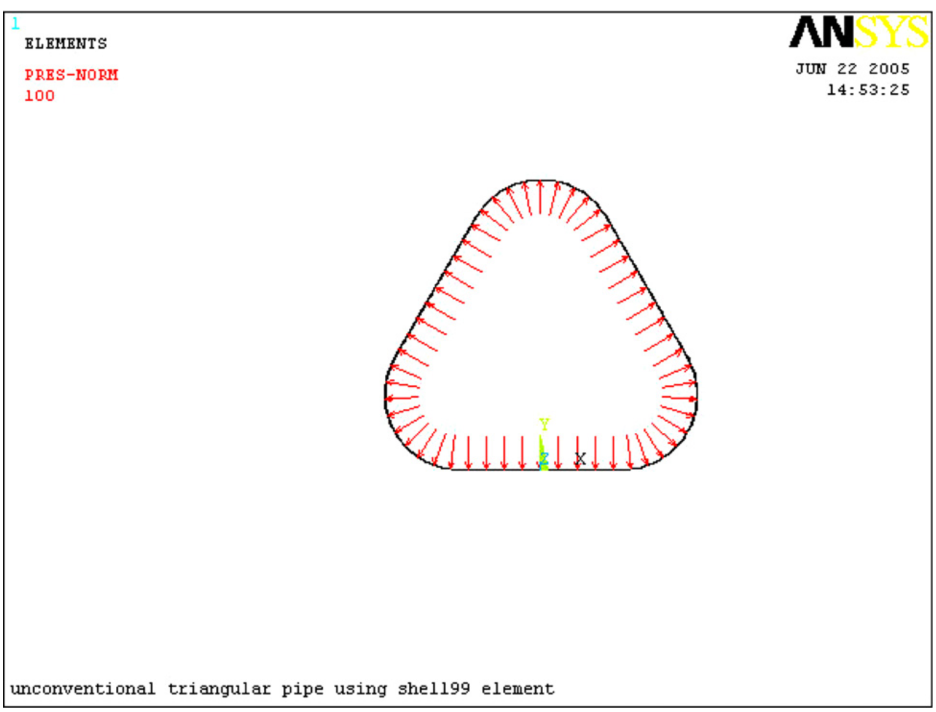

Fig. 6. Typical internal pressure direction in triangular pipe.

type, three different element types (shell-91, shell-99 and shell-181) are checked and shell-181 gives reasonably close results to analytical results. Shell-181 is a four-node element well-suited for linear, large rotation, and/or large strain non-linear applications. The convergence of the results were checked by changing element size from coarse to fine and then finest size was used in all cases. For failure analysis following allowable stress were used for this material: $\sigma_{\text{axial}} = 46.75$ MPa, $\sigma_{\text{hoop}} = 140.66$ MPa, $\tau_{xy} = 18$ MPa.

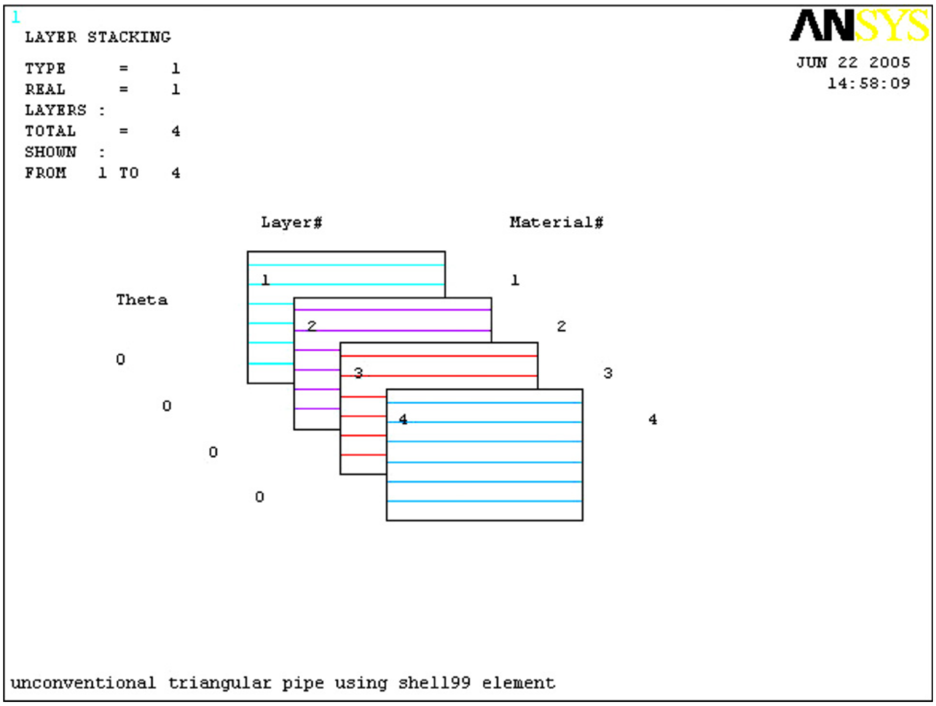

Fig. 7. Four layers stacking/orientation in shell-99 elements.

3. Results and discussion

In filament-winding a number of glass reinforcing strands are impregnated with resin and are then applied, under tension, to a mandrel. Repeated passes establish a layered-construction until the desired wall-thickness is achieved. The angles at which the glass fibers are oriented in reference to the axis of the mandrel may vary anywhere from 0° to 90°. Typical layers exhibit tensile and stiffness properties nearly 20 times higher in the direction of the

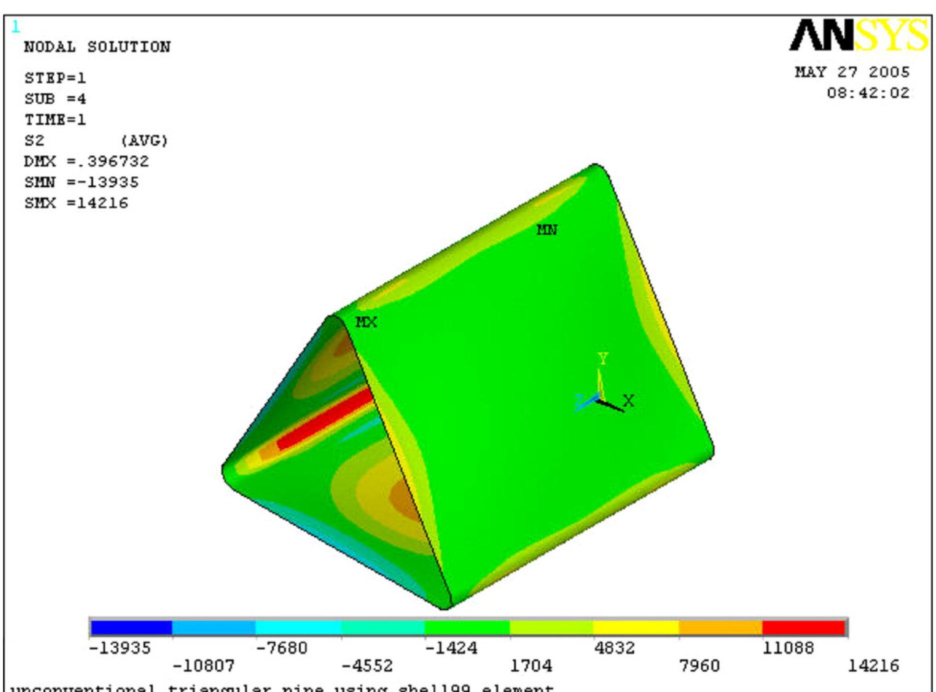

Fig. 8. Hoop stress distribution of a triangular shape pipe (fillet radius 12.5 mm).

fibers than in the direction perpendicular to the fibers. In this study the effects of fiber orientation, tension and corner fillet radius has been evaluated.

The effect of fillet radius on peak hoop stress is studied by changing the fillet radius from 12.5 mm to 50.0 mm. The hoop stress distributions for triangular-shaped pipes for fillet radius 12.5 mm and 50.0 mm are shown in Figs. 8 and 9. Similarly, in other shaped pipes the peak hoop stress have been investigated. The influence of fillet radius on peak hoop stress is shown in Fig. 10. As the fillet radius increases, the peak hoop stress decreases. With increasing fillet radius, the corner stress concentration decreases and stress is distributed to other regions. At fillet radius 50 mm the trend of the peak stress for triangular and square shape pipe is constant. The optimum fillet radius may be near to fillet radius 50 mm.

The variation of peak hoop stress with fiber orientation angle is shown in Fig. 11. The orientation angles used for four layers are 0°, 45°, 90° and (0°, 45°, 90°, 135°). The peak stress changes with fiber orientation. The optimum

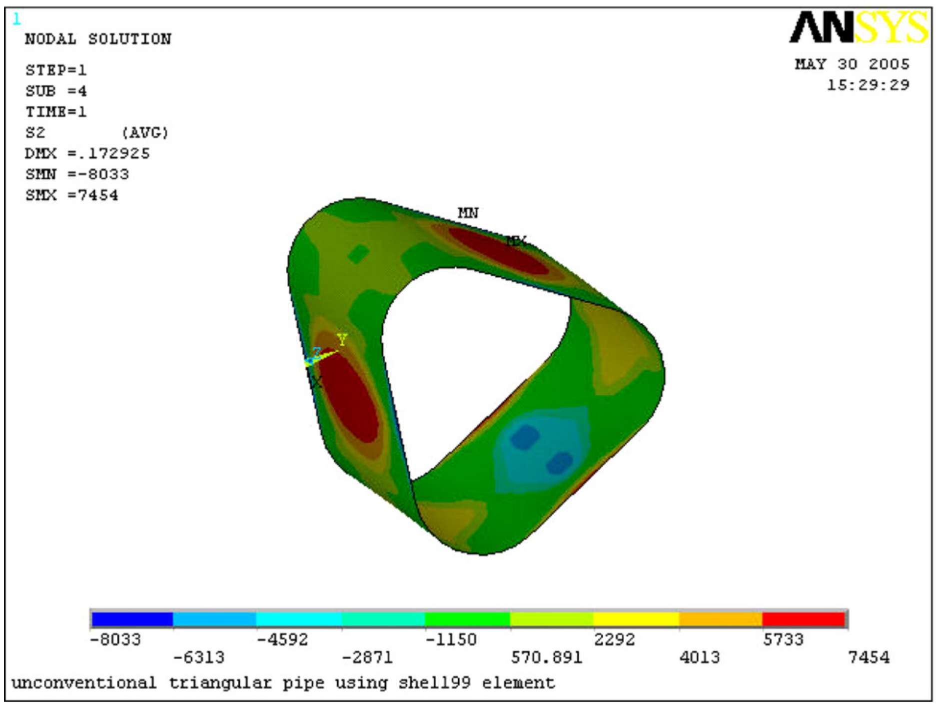

Fig. 9. Hoop stress distribution of a triangular shape pipe (fillet radius 50.0 mm).

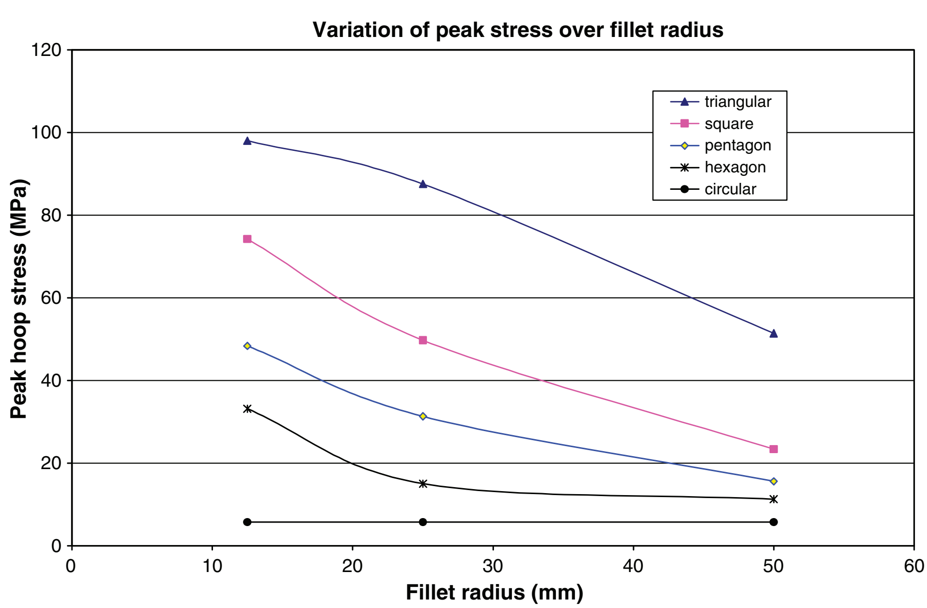

Fig. 10. Variation of peak stress versus fillet radius.

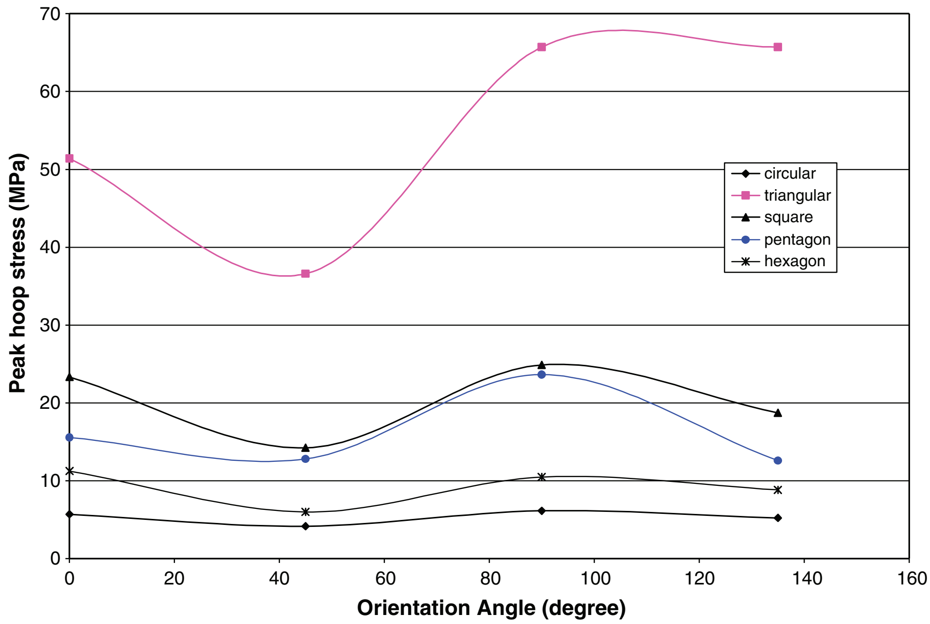

Fig. 11. Variation of peak hoop stress versus fiber orientation angle.

orientation angle may be between 45° and 60°. In the last case, four angles for four layers are used and peak stress decreases comparing to 0°, 90° stacking. But it is still higher than 45° orientation.

In failure analysis, maximum principal stress and Tsai–Wu stress theories are used. The applied internal pressure is increased and the maximum stress and Tsai–Wu failure criteria are checked. When the local stress is equal to maximum allowable stress, the failure pressure at that condition is recorded. The initial stress is also applied to the model. The failure pressure at different initial stress is shown in Fig. 12. With initial stress the failure pressure increases but the amount is not significant.

Figs. 13 and 14 show shear stress distribution for circular and square pipes, respectively under torsional loading of 11.3 N m about longitudinal axis. Similarly for other shapes, the peak shear stress distributions are plotted for different fillet radius. The comparison of peak shear stress versus fillet radius for different pipes is shown in Fig. 15. The peak shear stress for circular pipe is less comparing to other pipes. This indicates that corner notch has some effects on shear stress. In other shapes particularly square

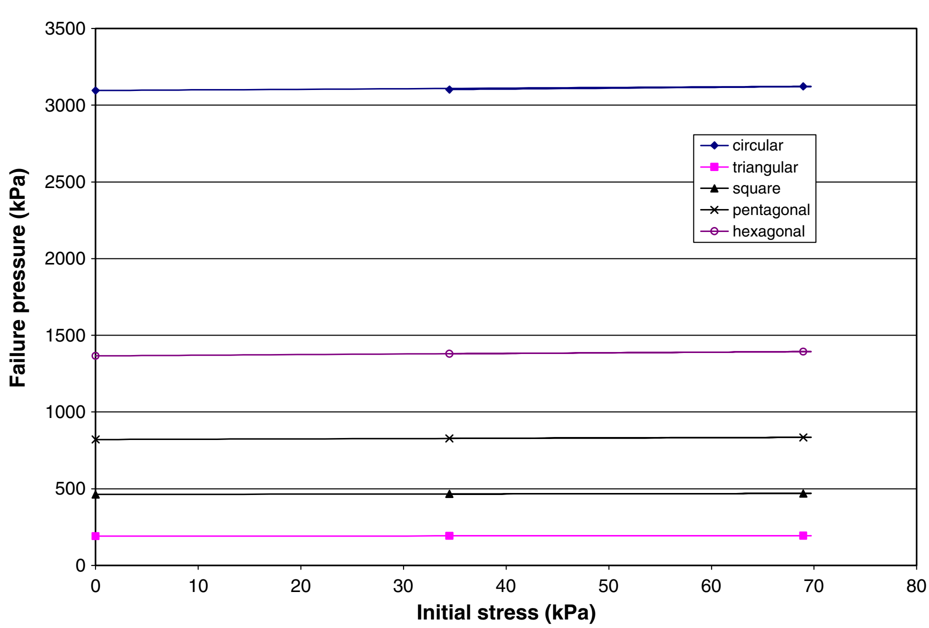

Fig. 12. Failure pressure at different initial stress.

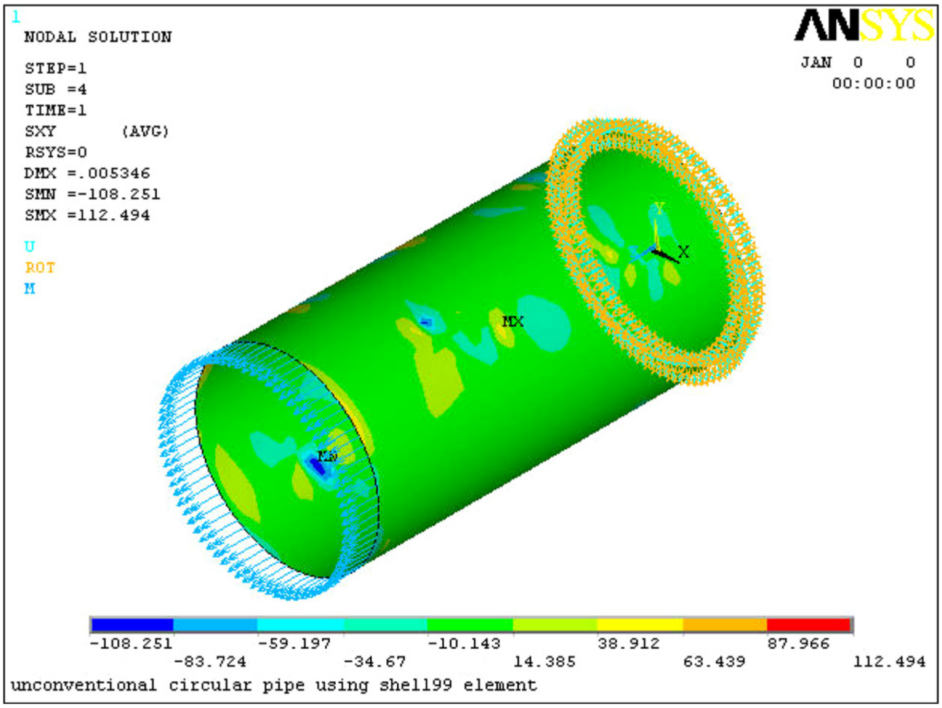

Fig. 13. Shear stress distribution of a circular pipe under torsion (11.3 N m) about longitudinal axis (z).

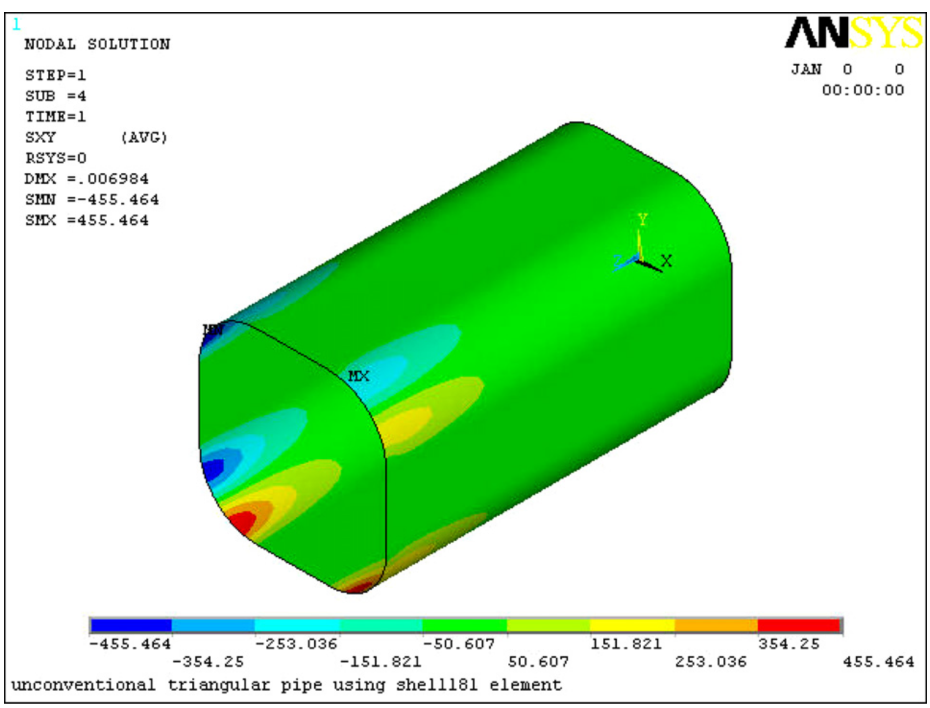

Fig. 14. Shear stress distribution of a square pipe under torsion (11.3 N m) about longitudinal axis (z).

pipe, the peak shear stress is less than all other pipes except circular. This may be due to the cross-sectional area variation. The triangular, square, pentagonal and hexagonal shapes are found from an inscribed circle of radius 75 mm. Therefore the periphery of the shapes are not equal and hence the cross-sectional areas are not same. For the area variation and notch corner the peak shear stress has been varied. For fillet radius 25.0 mm and 50 mm, the maximum peak shear stress is found in triangular pipe. But for fillet radius of 12.5 mm, the maximum peak shear stress is found in hexagonal pipe. The combined effect of fillet radius and variation in areas may be the crucial factor

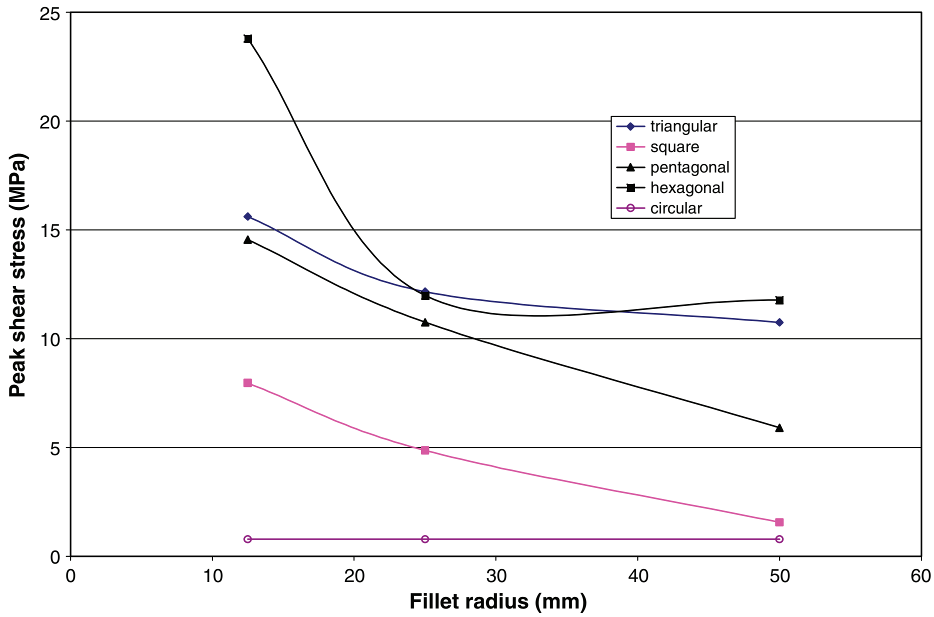

Fig. 15. Peak shear stress versus fillet radius for different pipes.

Table 1
Comparison of analytical and FEM results of torsional shear stress (kPa)

Pipe cross-sections	Analytical (kPa)	FEM (average) (kPa)
Circular	275.8	344.75
Square	882.56	1034.25

for shear stress variation. The circular and square pipes have also been analyzed using Eq. (4). The comparison of analytical and FEM results of shear stress is shown in Table 1. In FEM analysis the shear stress is found taking approximate average from the shear stress distribution. Therefore the variation is quite large.

4. Conclusions

The fillet radius, fiber orientation angle and initial stresses have effects on peak hoop stress. As the fillet radius increases the peak hoop stress decreases and optimum fillet radius may be ≈50 mm for this dimension (150 mm diameter, 304.8 mm length, and 6.35 mm thickness). The optimum fiber orientation lies in-between 50° and 60° angle. Initial stress has some marginal effects on failure pressure: i.e., it increases stiffness and consequently increases the failure pressure. The torsional peak shear stress for circular pipe is less than other pipes. The corner notch shape has some effect on shear stress and it increases shear stress.

Acknowledgements

The authors wish to acknowledge the support received from Louisiana Board of Regents ITRS Grant # 127-40-4166, EDO Fiber Science, Salt Lake, Utah (Grant # 127-40-6170) and the Department of Mechanical Engineering at Louisiana State.

References

[1] Zhao Y, Pang SS. Stress–strain and failure analyses of composite pipe under torsion. J Pressure Vessel Technol-Trans ASME 1995;117(3):273–8.

[2] Timoshenko S, Goodier JN. Theory of elasticity. 2nd ed. New York, USA: McGraw-Hill; 1951. p. 298–9.