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Studying Dynamic Equilibrium of Cloud Computing Adoption with Application of Mean Field Games

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Abstract— **Computing is undergoing a substantial shift from client/server to the cloud. The enthusiasm for cloud infrastructures is not only present in the business world, but also extends to government agencies. Managers of both segments thus need to have a clear view of how this new era will evolve in the coming years, in order to appropriately react to a changing economic and technological environment. In this study, we explore the dynamic equilibrium of cloud computing adoption through the application of Mean Field Games. In our formulation, each agent (i.e., each firm or government agency) arbitrates between "continuing to implement the traditional on-site computing paradigm" and "moving to adopt the cloud computing paradigm". To decide on his level of moving to the cloud computing paradigm, each agent will optimize a total cost that consists of two components: the effort cost of moving to the cloud computing paradigm and the adoption cost of implementing the cloud computing paradigm. In the formulation, the adoption cost is linked to the general trend of decisions on the computing paradigm adoption. Thus, an agent's optimal level of transition to the cloud computing paradigm is not only dependent on his own effort and adoption costs but also affected by the general trend of adoption decisions. The problem is solved by a system of partial differential equations (PDEs), that is, mean field games PDEs, which consists of a backward PDE, the Hamilton Jacobi Bellman equation for a controlled problem, and a forward Fokker-Planck equation transported by the optimal control from the backward HJB equation. Thus, the solution to the forward Fokker-Planck equation enables us to study the dynamic evolution of the density of the cloud computing adoption. It therefore allows us to investigate the impact of the general trend of technology adoption decisions on a firm's optimal decision of technology transition.**

I. Introduction

Computing is undergoing a significant shift from client/server to the cloud, a shift similar in importance and impact to the transition from mainframe to client/server. Speculation abounds on how this new era will evolve in the coming years, and managers in business, industry and government have a critical need for a clear vision of where the industry is heading. To answer the question of "How will the computing paradigm transition evolve in the coming years?", we believe that the general trend of decision has a big impact on individual decisions, as it is often the case in the evolution of information technology (IT). The objective of this work is to model and analyze this aspect.

Economics aspects are naturally essential in shaping industry transformations. Accordingly, the emergence of cloud services is fundamentally shifting the economics of IT in several economics aspects. First, cloud technology standardizes and pools IT resources and automates many of the maintenance tasks done manually today. Second, cloud architectures facilitate elastic consumption, self-service, and pay-as-you-go pricing. Third, cloud allows core IT infrastructure to be brought into large data centers that take advantage of significant economies of scale in three areas, supply-side savings, demand-side aggregation and multi-tenancy efficiency. It can thus be seen that a firm, shifting from the traditional on-site paradigm to the cloud computing paradigm, may benefit from cost advantages through either shifting fixed costs to variable costs or through paying lower unit operation costs as a result of the positive economy of scales that cloud service providers experience. Consequently, it is reasonable to analyze a firm's use of cloud computing as a cost minimization problem. However, individual costs of cloud computing adoption are also hinged on the general trend of adoption decisions of other entities as seen from the lower unit operation costs thanks to the positive economy of scales mentioned above. Focusing on such an economics perspective, we study the transition dynamics from the traditional on-site computing paradigm to the cloud computing paradigm as the dynamic equilibrium of decision on the technology upgrade through studying mean field games (MFG), introduced by Jean-Michel Lasry and Pierre- Louis Lions ([2]). MFG is well adapted to economic modeling and is devoted to the analysis of differential games with a large number of "small" players who have very little influence on the overall system.

Recent studies related to economics of cloud computing include Tak et. al. [6] and Kantarcioglu et. al. [5]. Tak et.al. [6] identify a comprehensive set of factors affecting the costs of a deployment choice, including in-house, cloud, and combination, and use "**NPV (Net Present Value)-based**" cost analysis for cloud computing adoption recommendations. They do not include the risk factor in their current version of analysis due to the complexity of quantifying associated security risk encountered with deployment choices. Kantarcioglu et. al. [5] use "**Real Options Theory**" to find the optimal time of shifting from the traditional on-site computing paradigm to the cloud computing paradigm. Different from Tak et. al. [6], Kantarcioglu et. al. [5] model the benefit arising from the computing paradigm adoption decision as a jump diffusion process, which captures both uncertainty and security impacts on the optimal decision of transition to the cloud computing paradigm. The optimal decision is determined by a threshold level. Depending on the specification of the model, the threshold level may be either identified as a single benefit value or identified as a relative benefit ratio from the adoption of two distinct computing paradigms.

In Kantarcioglu et. al. [5], the recommendation of computing paradigm adoption is a binary rule, either moving to the cloud computing paradigm or remaining the use of the traditional on-site computing paradigm. The current study, through studying MFG, focuses on the impact of the general trend of decision on the choice of computing paradigm transition times. The result is a dynamic equilibrium of the level of cloud computing adoption, replacing the stopping time decision (i.e., a binary decision) provided in Kantarcioglu et. al. [5]. That is, through the current study, we arrive at a firm's optimal level of use of cloud computing, which can be any level between zero adoption of the cloud computing paradigm and complete adoption of the cloud computing paradigm, considering the general trend of adoption decisions.

The remainder of the paper is organized as follows. In Section II, we first provide a general framework of modeling such computing technology transition problems, followed by giving a specific formulation of the problem. The existence of solution of the specific model formulation proposed is also studies. In Section III, we discuss the challenges in implementing the proposed model. We present concluding remarks in Section IV.

II. The Model

We aim at characterizing the dynamic equilibrium of the computing paradigm shift between the choice of the traditional on-site computing paradigm and the cloud computing paradigm. We look at a large economy in a continuous time setting so that there is a sufficiently large number of firms, allowing an averaging effect.

A. General Framework

The formulation focuses on the impact of the general trend of decisions on the choice of technology transition times. In such a setting, a continuum of agents having homogenous preferences, pay a cost to move from one point to another point in the state space, X, where $X(t) \in [0,1]$ represents the level of use of cloud computing technology. Agents have the common cost function $f(t, \alpha, x, m(\cdot),)$ representing what an agent pays to have the characteristics x (i.e., the level of cloud computing adoption) at time t. $m(t,\cdot)$ is the density of the population for a given level of x and $\alpha(t)$ is a control variable modeling the effort of the firm in increasing its use of cloud computing technology. The evolution of $X(t)$ is described as a random process with a drift term $\ell(x,\alpha)$ and a diffusion term $\sigma(x)$; some specific formulation is given below in subsection II-B.

Each agent makes his optimal decision on the use of cloud computing by minimizing the associated cost. Therefore, the problem of each agent is written as the following cost minimization control problem:

$$\begin{cases} u(t,X) \\ = \inf_\alpha \mathrm{E}\left[\int_0^T f(\alpha(t), X(t), m(t,\cdot))e^{-\mu t}dt\right] \\ dX_t = \ell(t, X(t), \alpha(t,))dt + \sigma(X(t))dW_t,\ X_0 = x \end{cases} \tag{1}$$

- W_t is a standard Brownian motion.
- μ is a discount rate.

Mean Field Games Approach

One solves (1), the problem of representative agent, for fixed $m(t,\cdot)$. The density is defined at the equilibrium as the probability density of the optimal $\widehat{X}(t)$. The control problem defined in (1) is equivalent to the following system of PDEs:

$$\begin{cases} \text{(HJB)} \qquad \partial_t u + \frac{\sigma^2}{2}\triangle u + H(x,m,Du) - \mu u = 0 \\ \text{(Fokker-Planck)} \qquad \partial_t m + \nabla(mq(\nabla u)) = \frac{\sigma^2}{2}\triangle m \end{cases}$$

with $m(0,\cdot)$ given, $u(T,\cdot) = 0$ and $m(t,\cdot)$ is a probability density function $\forall\ t$ where $H(x, m(\cdot), q) = \inf_\alpha[f(\alpha, x, m(\cdot)) + q \cdot \ell(x,\alpha)]$.

Thus, by MFG, the solution to the optimal control problem leads to a system of HJB and Fokker-Planck equations.

B. Specific Formulation in Application

In what follows, we propose specific formulations of the state process, $X(t)$ and the cost function for the potential application of the computing paradigm transition choice between the traditional on-site computing paradigm and the cloud computing paradigm.

1) EVOLUTION OF THE OPERATION MODE - LEVEL OF USE OF THE CLOUD COMPUTING PARADIGM: We associate to a given firm a degree of cloud computing adoption in its current operation mode, $X_0 = x \in [0,1]$, which fully characterizes its initial state. In such a formulation, $x = 0$ corresponds to a firm which has not yet moved its operation mode to the cloud computing, whereas $x = 1$ indicates a firm fully operating on the cloud computing paradigm. The initial density of firms is given by $m(0,\cdot)$, which describes firms' degree of operation dependence on the cloud computing paradigm at time 0, and corresponds to the density of initial variable X_0. Each firm makes its decision between the adoption of the on-site computing paradigm and the cloud computing paradigm by exercising its level of effort of α. Therefore, the dynamics of a firm's state evolves according to the following controlled diffusion process:

$$dX_t = \alpha_t(1 - X_t)dt + \sigma X_t(1 - X_t)dW_t,\ X_0 = x \tag{2}$$

In (2),

- α_t, the control variable, is the effort required to move to cloud computing paradigm.

- W_t is a standard Brownian motion, and σ is the noise from the cloud computing paradigm.

Equation (2) guarantees the process X stays in $[0, 1]$, and it makes economic senses in that:

- When a firm fully transits to the cloud computing paradigm, no effort can be contributed to the computing paradigm shift, and, meanwhile, no disturbances occurs as to the level of cloud computing adoption since the transformation has been completed.
- When a firm has not yet operated on the cloud computing paradigm, there would be no noise from the level of cloud computing adoption.

2) COST FUNCTIONS: We consider that a firm's decision of shifting from the traditional on-site computing paradigm to the cloud computing paradigm incurs two costs, *the effort cost* and *the adoption cost*. In what follows, we give detailed descriptions of these two costs with two distinct cost functions:

- *Effort Cost*: This cost measures the cost of the effort that a firm has to make when it decides to move its IT adoption from the traditional onsite paradigm to the cloud computing paradigm. Such costs may include the effort that a firm has to commit to necessary infrastructure changes due to the IT shifts. Mathematically, it is the cost to change the state, i.e., the operation mode (the level of use of cloud computing). We model the effort cost by taking the quadratic form in the effort, the control variable, since it is rational to assume that such a cost increases at an increasing rate with respect to the effort required to make for moving the size of the state. The larger the size of the state that a firm would like to move at a given time, the larger the effort the firm required to make, thus the associated cost. That is, we have the effort cost specified as:

$$h(\alpha_t) := \frac{\alpha_t^2}{2} \tag{3}$$

- *Cloud Computing Adoption Cost*: One significant economic gain of cloud computing adoption is from the cost advantage. The characteristics of pay-per-service fees charged to the cloud service users releases a firm from investing a significant amount of fixed cost in setting up its own computing paradigm, instead a firm only has to pay what it uses in its production/service process, which is in essence a variable cost. The other cost advantage is caused by the positive economy of scale that cloud service providers experience when producing cloud services. Due to the effect of this positive externality, the pay-per-service fee charged by the cloud service provider shall be lower as more firms move to the cloud computing paradigm. Considering the above-mentioned characteristics and motivated by Aim'e Lachapelle and Jean-Michel Lasry [3], we specify the cost of cloud computing adoption as:

$$g(t, x, m) := \frac{cx}{1 + \gamma m(t, x)} + p_O(t) \times (1 - x) \tag{4}$$

where c and γ are positive numbers, $p_O(t)$ measures the per unit rate of operation in the traditional on-site paradigm at time t, and $m(t, x)$ is the density of firms at time t. We note that:

  - The linear term cx in (4) describes the characteristics of pay-per-service fee of using cloud computing services since it indicates that the higher the degree of a firm's operation utilizing the cloud computing paradigm, the higher the cloud adoption cost that a firm will have to pay due to larger amount of service usage.
  - The term $p_O(t) \times (1 - x)$ measures a firm's cloud computing adoption cost in the sense that, for its business segments of operation modes that have not been shifted to the cloud computing paradigm, a firm has to continue paying the cost of operating on the traditional on-site computing paradigm.
  - The term $\gamma m(t, x)$ shown in the denominator of (4) depicts the impact of the positive externality (i.e. the positive economy of scale) of the cost that cloud service providers experience when producing cloud services. The term γ characterizes the positive cost externality that occurs to cloud service providers when they have to produce services in large quantity arising from large demands. Thus, the overall positive economy of scale of cost that cloud service producers experience shall be positively related to the number of cloud service users. We characterize the number of cloud service users by the density of the cloud service users. As a result, we specify this relation by a positive linear relation term $\gamma m(t, x)$. The positive externality of the cost that cloud service providers experience shall result in lowering the pay-per-service fee charged to the cloud service user, and therefore, we model this as an inverse relation by $\frac{1}{1 + \gamma m(t, x)}$.

  From (4), we observe that the more firms move to cloud computing, the larger the adoption cost related to cloud computing decreases. The reason is that, as more firms move to use cloud computing services, cloud service providers experience larger positive externality of production cost, and thus lower the pay-per-service fee charged to cloud service users, reflected by the term $\frac{1}{1 + \gamma m(t, x)}$. Therefore, firms should do the same choice, that is, to stay together to the choice of computing paradigm. Additionally, the presence of the dependence of the adoption cost on the density of firms describes the situation that the general trend of technology adoption decisions may have a big impact on individual decisions, which we aim at studying and modeling.

3) OBJECTIVE FUNCTION: Our main objective is to study a firm's optimal level of use of cloud computing. With the cost proposed in subsection II-B.2, each rational firm makes its decision on the level of transition to the cloud computing paradigm to minimize its expected discount cost with respect to the effort cost, α_t. That is, we an write it as the following control problem:

$$\begin{cases} u(t,X) \\ = \inf_\alpha \mathrm{E}\left[\int_0^T (h\left(\alpha\left(t,X_t^x\right) + g\left(t,X_t^x,m\left(t,X_t^x\right)\right)\right))e^{-\mu t}dt\right] \\ dX_t = \alpha_t(1-X_t)dt + \sigma X_t(1-X_t)dW_t,\ X_0 = x \end{cases} \tag{5}$$

where μ is a discount rate.

4) THE SOLUTION: We discuss the solution to the control problem defined in (5) as well as the solution to the deterministic case (i.e., $\sigma = 0$ in (2)). Moreover, we study the existence of the solution with the technique and theorem used in Lachapelle et. al. [4].

4-1) THE MEAN FIELD GAMES PDEs

Proposition 1: The control problem defined in (5) is a solution to the following system equations:

$$\begin{cases} \partial_t u(t,x) + \frac{\sigma^2}{2}x^2(1-x)^2\partial_{xx}^2 u(t,x) - \mu u(t,x) \\ \qquad + \frac{(g(t,x,m)+\partial_x u(t,x))^2}{2} + g(t,x,m) \\ \qquad + (g(t,x,m) + \partial_x u(t,x))\partial_x u(t,x) = 0 \\ \partial_t m(t,x) - \frac{\sigma^2}{2}\partial_{xx}^2[x^2(1-x)^2 m(t,x)] \\ \qquad - \partial_x((g(t,x,m) + \partial_x u(t,x))m(t,x) = 0 \end{cases} \tag{6}$$

with $m(0,\cdot) = m_0$, and $u(T,\cdot) = 0$.

Proof: The Hamilton Jacobi Bellman equation associated to the optimal control problem described in (5) is:

$$\partial_t u(t,x) + \frac{\sigma^2}{2}x^2(1-x)^2\partial_{xx}^2 u(t,x) - \mu u(t,x) + \min_\alpha\{h(\alpha) + g(t,x,m) + \alpha\partial_x u(x,t)\} = 0 \tag{7}$$

$m(t,x)$ is transported by the optimal effort decision of moving to the cloud computing, denoted by $\hat{\alpha}(t,x)$; $m(t,x)$ is the solution of the Fokker-Planck equation:

$$\partial_t m(t,x) - \frac{\sigma^2}{2}\partial_{xx}^2[x^2(1-x)^2 m(t,x)] + \partial_x(\hat{\alpha}(t,x)m(t,x)) = 0 \tag{8}$$

with $m(0,\cdot)$ given.

m is linked to u by the optimal control of HJB equation, this optimal control is given by

$$\hat{\alpha}(t,x) = -g(t,x,m) - \partial_x u(t,x) \tag{9}$$

u depends on m through $g(t,x,m)$.

The coupled equations associated to the optimization problem are

$$\begin{cases} \partial_t u(t,x) + \frac{\sigma^2}{2}x^2(1-x)^2\partial_{xx}^2 u(t,x) - \mu u(t,x) \\ \qquad + \frac{(g(t,x,m)+\partial_x u(t,x))^2}{2} + g(t,x,m) \\ \qquad + (g(t,x,m) + \partial_x u(t,x))\partial_X u(t,x) = 0 \\ \partial_t m(t,x) - \frac{\sigma^2}{2}\partial_{xx}^2[x^2(1-x)^2 m(t,x)] \\ \qquad - \partial_x((g(t,x,m) + \partial_x u(t,x))m(t,x)) = 0 \end{cases} \tag{10}$$

with $m(0,\cdot) = m_0$, and $u(T,\cdot) = 0$. ■

Proposition 2: In the deterministic case, i.e., $\sigma = 0$, Mean Field Games PDEs in (6) reduces to:

$$\begin{cases} \partial_t u(t,x) - \mu u(t,x) \\ \qquad + \frac{(g(t,x,m)+\partial_x u(t,x))^2}{2} + g(t,x,m) \\ \qquad + (g(t,x,m) + \partial_x u(t,x))\partial_x u(t,x) = 0 \\ \partial_t m(t,x) - \partial_x((g(t,x,m) + \partial_x u(t,x))m(t,x)) = 0 \end{cases} \tag{11}$$

with $m(0,\cdot) = m_0$, and $u(T,\cdot) = 0$

Proof: The proof is similar to the proof of Proposition 1. ■

4-2) THE EXISTENCE OF THE SOLUTION

The study of the existence of the solution to the system of PDEs in (6) is similar to and follows Lachapelle et. al. [4]. Thus, we give a sketch of the steps.

- Consider the space domain $(0,1)$ (denoted as Ω) with Neumann boundary conditions and the d-dimensional thorus $\mathbb{T}^d$.
- Define the time-space domain $Q := [0,T] \times \Omega$, a nonnegative continuous function $\eta : Q \times \mathbb{R} \to \mathbb{R}_+$, and the function:

  $$\Phi : \mathrm{M}_b^{ac}(\mathbb{R}_+) \times [0,T] \to \mathbb{R}$$
  $$(m,t) \mapsto \Phi(m)(t) := \int_\Omega \eta(t,x,m(x))dx$$

  where $\mathrm{M}_b^{ac}(\mathbb{R}_+)$ denoted the space of Borel measures supported on Ω, absolutely continuous with respect to the Lebesgue measure.
- The control problem in (5) can be written as the following Eulerian formulation:

  $$\begin{cases} \inf_\alpha \int_0^T [\int_\Omega \frac{\alpha(t,x)^2}{2} m(t,x)dx + \Phi(m_t)(t)]dt \\ \partial_t m - \frac{\sigma^2}{2}\triangle m + \mathrm{div}(\alpha m) = 0,\ m(0,\cdot) = m_0, \end{cases} \tag{12}$$

  with boundary conditions. Note that here $\Phi(m_t)(t) := \int_0^1 g(t,x,m)m(t,x)dx$.
- Assuming $\frac{\sigma^2}{2} = 1$ and introducing the transformation $q = \frac{\alpha}{2}m$ i.e., the functions

  $$\Psi(a,b) := \begin{cases} \frac{|a|^2}{b} \text{ if } (a,b) \in \mathbb{R}^d \times \mathbb{R}_+^* \\ +\infty \text{ otherwise} \end{cases}$$

  and

  $$K(q,m) := \int_\Omega \Phi(q,m) + \int_0^T \Phi(m_t)(t)dt.$$

  In fact, we can then write

  $$K(q,m) = \begin{cases} \int_0^T \left[\left(\int_\Omega \frac{|q|^2}{m}\right) + \Phi(m_t)(t)\right]dt \\ := \int_0^T \left[\int_\Omega \frac{I\alpha(t,x)|^2}{2} m(t,x)dx \right. \\ \qquad \left. + \Phi(m_t)(t)\right]dt \text{ if } q = \frac{\alpha}{2}m \\ +\infty \text{ otherwise} \end{cases}$$

Now (12) can be written as

$$\begin{cases} \inf_{(q,m)\in B} K(q,m) \\ B := \{(q,m) : q \in L^2(Q),\ m \text{ weak solution of} \\ \partial_t m - \triangle m + \operatorname{div}(q) = 0,\ m(0,\cdot) = m_0(\cdot) \in L^2(\Omega), \\ m \in W(0,T)\}. \end{cases}$$

- Introduce the penalized problem

$$\inf_{(q,m)\in B} K_\epsilon(q,m) := K(q,m) + \epsilon ||q||_2^2 \tag{13}$$

where $||.||_2$ denotes the $L^2(Q)$-norm.
- Make use of the Theorem 2.1 in Lachapelle et. al. [4]. The theorem states:
If $m_0 \in L^2(\Omega)$, then the minimization problem of (13) admits a solution $(q_\epsilon, m_\epsilon) \in B$ with $q_\epsilon \in L^2(Q)$. Moreover,
$\lim_{\epsilon\to 0} \left(\min_{(q,m)\in B} K_\epsilon(q,m) \right) = \inf_{(q,m)\in B} K(q,m).$

III. Comments and Discussion

The formulation proposed in subsection II-B leads us to the result that the representative agent's optimal level of use of cloud computing is solved by a system of partial differential equations, that is, mean field games PDEs, which consists of a backward PDE, the Hamilton Jacobi Bellman equation for a controlled problem, and a forward Fokker-Planck equation transported by the optimal control from the backward HJB equation. The solution to the forward Fokker-Planck equation enables us to study the dynamic evolution of the density of the cloud computing adoption.

The Mean Field PDEs in (6) have to be solved numerically. Given m_0, X_0, σ, c, γ, and $p_O(t)$, the evolution of m can be obtained. Therefore, the equilibrium of computing paradigm transition can be analyzed. However, in order to obtain practically useful insights of the current trend of the choice between the traditional on-site computing paradigm and the cloud computing paradigm, the following challenging issues need to be overcome practically to assure the good fitness of the model in the practice:

- An appropriate approach of identifying the initial probability density function, m_0, must be proposed.
- The measurement of the noise, σ, arising from the cloud computing adoption has to be appropriately addressed and proposed.
- A sound and agreed-upon measurement of the positive externality, γ, related to the cost of cloud service providers must be characterized.
- The consistent estimates of the per unit rate of operation employing the traditional on-site paradigm, $p_O(t)$, should be recommended

In addition, Gueant (2008) [1], who seeks to build a stationary solution with the model studied, provides certain formulation of cost functions which leads to explicit closed form solutions to the proposed mean field games PDEs. The benefit of this is that the exact analysis can be performed in contrast to the numerical approximation. In addition, the comparative statistical analysis can be easily performed and analyzed. Motivated by this, we are trying some other potential forms of cost functions related to cloud computing adoption for the purpose of arriving at the explicit closed form solution. Without doubts, the potential candidates of the cost functions have to be justifiable in practice.

IV. Conclusion

As observed in most cases of choices of technology upgrades made by individuals, we believe that the general trend of decision has a big impact on individual decisions in making the choice between continuing the use of the traditional on-site computing paradigm and switching into the adoption of the cloud computing paradigm. Therefore, it is important to take into account the general trend of use of cloud computing in order to make a good decision. Thanks to MFS, we are able to model and analyze the question of interest in such an aspect.

The use of MFG approach for the study of the computing paradigm transition is a tradeoff between the main drawback and the main advantage associated with the properties of thus approach. The main drawback is that the homogeneity of decision makers is assumed in this model. The main advantage of using MFG approach is the simplification thanks to the averaging effect of the decision of the individual agent.

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